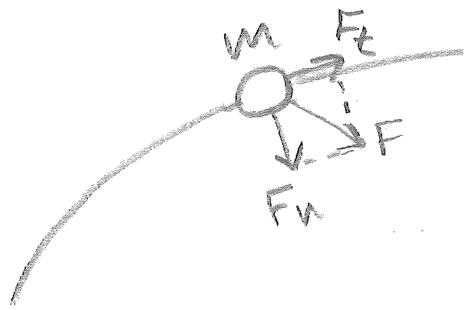
Före läsning 16 23/02-15

Enerilagar

Partikel



$$F_t = ma_t = m\ddot{s}$$

$$\left(F_n = ma_n = m\frac{\dot{s}^2}{\rho}\right)$$

$$\ddot{s} = \frac{d}{dt}(\dot{s}) = \frac{dv}{dt}$$

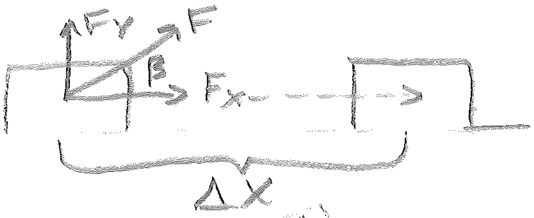
$$F_t = m\frac{dv}{dt}\cdot\frac{ds}{ds} = m\frac{dv}{ds}v$$

$$\Rightarrow \underbrace{\int_{s_1}^{s_2} F_t\,ds}_{\text{Arbete}} = \int_{v_1}^{v_2} \underbrace{mv}_{\text{Rörelsemängd}}\,dv = \underbrace{\frac{mv_2^2}{2} - \frac{mv_1^2}{2}}_{\text{Ändring av rörelseenergi (Kinetisk energi) } T}$$

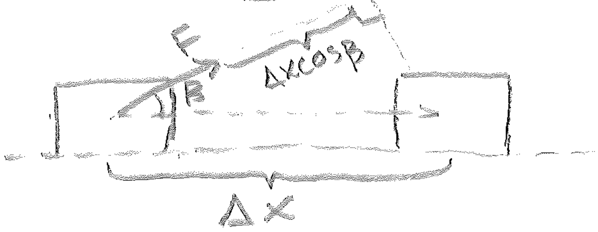
(*) $U_{1-2} = T_2 - T_1$

Lagen om den kinetiska energin

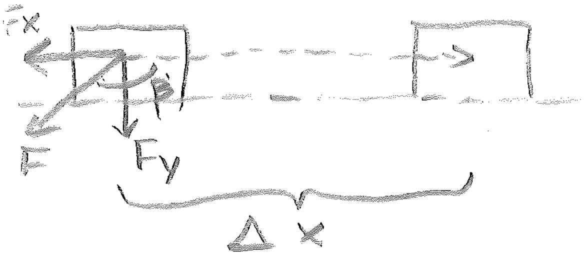
Arbete:



$U = F_x \Delta x = F\cos\beta\,\Delta x$



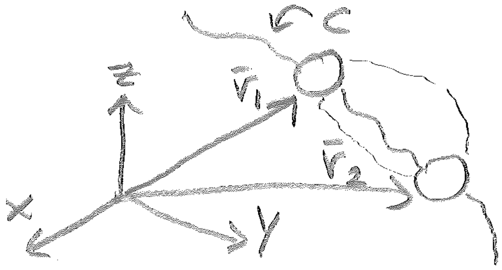
$U = F\Delta x\cos\beta$



$U = F\cos\beta\,\Delta x$, Ger nu negativt värde, $\beta > 90^\circ$

$$(*) \quad dU = \bar{F} \cdot d\bar{r}$$

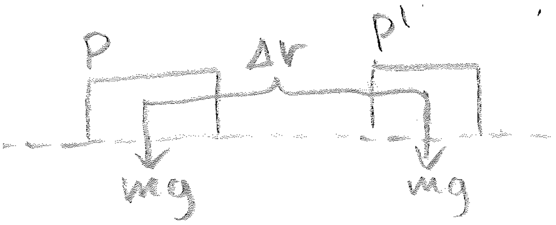
$$U = \int_{\bar{r}_1, C}^{\bar{r}_2} dU = \int_{\bar{r}_1, C}^{\bar{r}_2} \bar{F} \cdot d\bar{r}$$



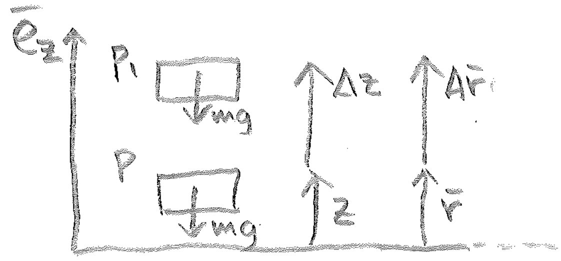
$$U = \int_{\bar{r}_1, C}^{\bar{r}_2} (F_x dx + F_y dy + F_z dz) =$$

$$= \int_{\bar{r}_1, C}^{\bar{r}_2} (F + ds) = \int_{\bar{r}_1, C}^{\bar{r}_2} (F_r dr + F_\theta d\theta + F_z dz)$$

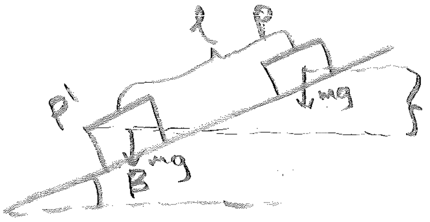
Tyngdkraftens arbete



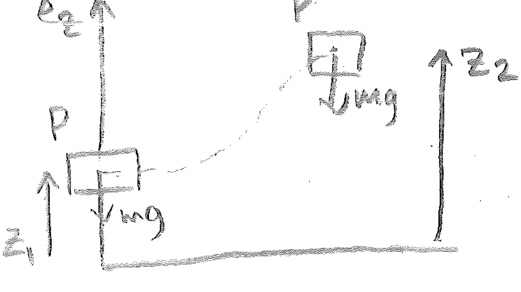
$U = 0$



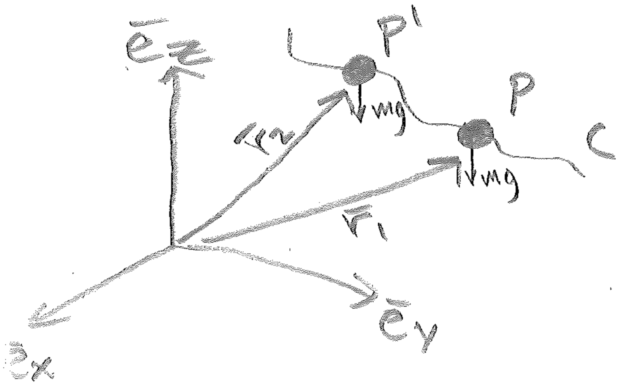
$$U = -mg\bar{e}_z \cdot \Delta\bar{r} = -mg\bar{e}_z \cdot \Delta z \cdot \bar{e}_z =$$
$$= -mg\Delta z$$



$\ell \sin\beta \quad U = mg\ell\sin\beta$



$U = -mg(z_2 - z_1)$

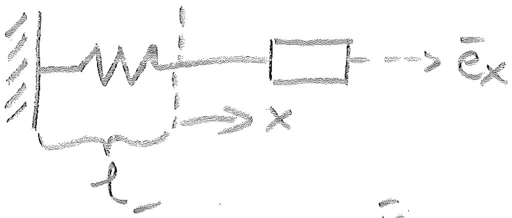


$$U = \int_{\bar r_1,\,C}^{\bar r_2} \bar F \cdot d\bar r = \quad -mg\,\bar e_z \quad dz\,\bar e_z$$

$$= \int_{\bar r_1,\,C}^{\bar r_2} (\underbrace{\bar F_x}_{0} d x + \underbrace{\bar F_y}_{0} dy + \bar F_z d z) =$$

$$= \int_{z_1}^{z_2} (-mg)\, dz = -mg(z_2 - z_1)$$

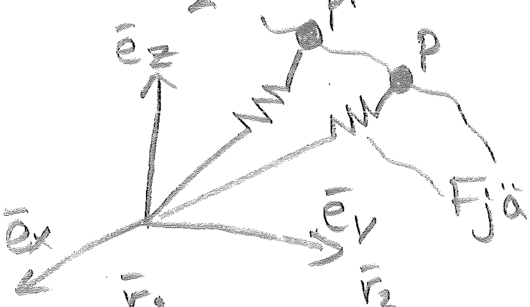
Fjäderkraftens arbete



ℓ: Naturlig längd på fjärdern
k: Fjäderkonstant
Linjär fjäder: $\bar F = -kx\bar e_x$

$$U = \int_{\bar r_1,\,C}^{\bar r_2} \bar F \cdot dr = \int_{\bar r_1,\,C}^{\bar r_2} (-kx\,dx + 0\,dy + 0\,dz) = -\int_0^x k\hat x\, d\hat x =$$

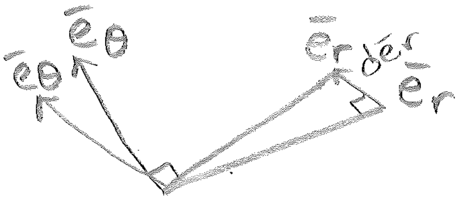
$$= -\frac{1}{2}kx^2$$



$$\bar F = -k(r-\ell)\bar e_r$$

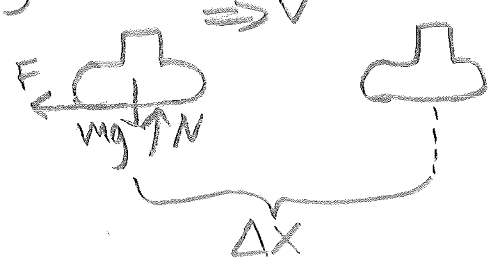
$$U = \int_{\bar r_1,\,C}^{\bar r_2} \bar F \cdot d\bar r = \int_{\bar r_1,\,C}^{\bar r_2} -k(r-\ell)\bar e_r \cdot d\bar r = -\int_{\ell}^{r} k(r-\ell)\, dr = -\frac{1}{2}k(r-\ell)^2$$

$$\left[\begin{array}{l} \bar r = r\bar e_r,\quad d\bar r = dr\,\bar e_r + r\,d\bar e_r \\ \bar e_r \cdot d\bar r = dr\,\underbrace{\bar e_r \cdot \bar e_r}_{1} + r\,\underbrace{\bar e_r \cdot d\bar e_r}_{=0} \end{array}\right.$$



Friktionskraftens arbete

Curlingklot



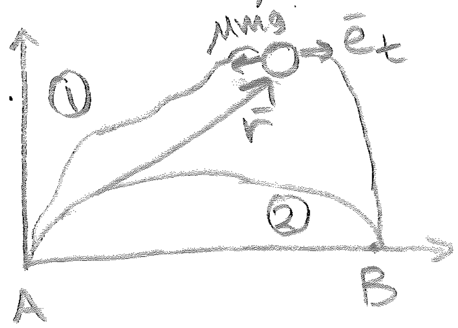
Vid glidning: $F = \mu_k N$

Här: $\mu_k = \mu$

$U = -F\Delta x = -\mu N \Delta x =$

$= (\Sigma F_y = m a_{Gy}) = -\mu m g \Delta x$

• Sträv yta sedd uppifrån:



$$U = \int_{\bar{r}_1}^{\bar{r}_2}{}_{c} \bar{F}\cdot d\bar{r} = \int_{S_A}^{S_B}{}_{c} F_t\, ds = \int_{S_A}^{S_B} (-\mu m g)\, ds =$$

$$= -\mu m g (S_B - S_A) = -\mu m g L$$

L: bankurvans längd

Vägberoende!

Konservativ kraft: Oberoende av vägen

Icke konservativa krafter: Vägberoende (friktion)

Fler ex, se s. 80 [210]

Potentiell energi

Varje konservativ kraft motsvaras av en potentiell energi V enligt:

$$V = \int_{ref}^{\bar{r}} dV = -\int_{ref}^{\bar{r}} \bar{F}\cdot d\bar{r}$$ ← Fix, men godtycklig referenspkt.

Definition potentiell energi: $dV = -\bar{F}\cdot d\bar{r}$

Definition arbete: $dU = \bar{F}\cdot d\bar{r}$

$\Rightarrow dV = -dU$

Kombinera med lagen om kinetiska energin:

$U_{1-2} = T_2 - T_1$ Småförändringar: $dU = dT$

$\Rightarrow dV = -dT \Rightarrow V - V_0 = -(T - T_0) \Rightarrow$ (*) $T + V = T_0 + V_0$

Mekaniska energilagen

Om alla krafter är konservativa!