8/10 2012 (14)

Extentagenomgång 24/8-2010

1. $\vec{E}(\vec{r},t) = E_0 \left\{ \cos(k_0 y - \omega t)\frac{\hat{x}+\hat{z}}{\sqrt{2}} + \sin(k_0 y - \omega t)\frac{\hat{x}-\hat{z}}{\sqrt{2}} \right\}$

Plana vågor har funktionsberoendet $\vec{E}(\vec{r},t) = \mathrm{Re}\left\{ \underset{\text{komplex}}{\vec{E}_0} e^{i(\vec{k}\cdot\vec{r}-\omega t)} \right\}$

a) Utbredningsriktning? $\hat{k} = \hat{e} = \frac{\vec{k}}{k}$

Från uppgiften $\vec{k} = k_0 \hat{y}$ $\Rightarrow$ $\boxed{\hat{e} = \hat{k} = \frac{\vec{k}}{k} = \hat{y}}$

Kontroll: $\vec{E}(\vec{r},t)\cdot\hat{k} = 0$.

b) Utnyttja att $\{\vec{E}, \vec{H}, \hat{k}\}$ är högersystem.

$\vec{H}(\vec{r},t) = \frac{1}{\eta_0}\hat{k}\times\vec{E}(\vec{r},t) =$ (Faradays lag)

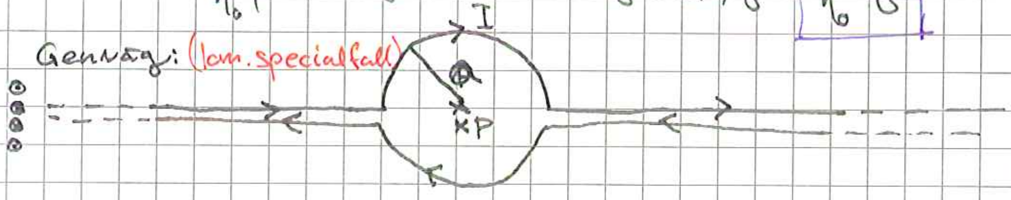
$= \frac{E_0}{\eta_0}\hat{y}\times\left\{ \cos(k_0 y - \omega t)\frac{\hat{x}+\hat{z}}{\sqrt{2}} + \sin(k_0 y - \omega t)\frac{\hat{x}-\hat{z}}{\sqrt{2}} \right\} = \boxed{\frac{E_0}{\eta_0}\left\{ \cos(k_0 y - \omega t)\frac{-\hat{z}+\hat{x}}{\sqrt{2}} + \sin(k_0 y - \omega t)\frac{-\hat{z}-\hat{x}}{\sqrt{2}} \right\}}$

c) Poyntings vektor: $\vec{S}(\vec{r},t) = \vec{E}(\vec{r},t)\times\vec{H}(\vec{r},t) =$

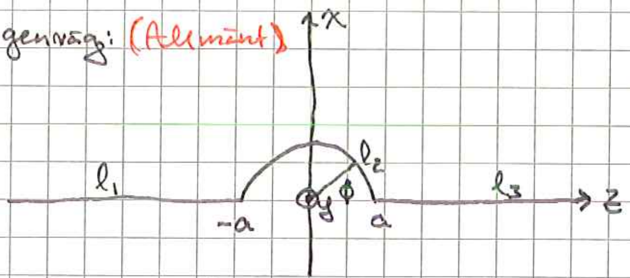
$= \frac{E_0^2}{\eta_0}\left\{ \cos^2(k_0 y - \omega t)\frac{\hat{x}+\hat{z}}{\sqrt{2}}\times\frac{(-\hat{z}+\hat{x})}{\sqrt{2}} + \sin^2(k_0 y - \omega t)\frac{\hat{x}-\hat{z}}{\sqrt{2}}\times\frac{(-\hat{z}-\hat{x})}{\sqrt{2}} \right\} =$

$= \frac{E_0^2}{\eta_0}\left\{ \cos^2(k_0 y - \omega t) + \sin^2(k_0 y - \omega t) \right\}\hat{y} = \boxed{\frac{E_0^2}{\eta_0}\hat{y}}$

2. Genväg: (om. specialfall)



Ingen genväg: (Allmänt)



Parameterframställ kurvan

l_1: $\vec{r}'(t) = t\hat{z}$ $\quad t\in[-\infty,-a]$

l_2: $\vec{r}'(t) = a\cos\phi\,\hat{z} + a\sin\phi\,\hat{x}$, $\phi\in[\pi,0]$

l_3: $\vec{r}'(t) = t\hat{z}$ $\quad t\in[a,\infty]$

Biot-Sawarts lag i origo $\vec{r} = \vec{0}$

$\vec{B}(\vec{0}) = -\frac{\mu_0}{4\pi} I \int \frac{d\vec{r}'\times\vec{r}'}{|\vec{r}'|^3}$ Från l_1 och l_3 inget bidrag ty $d\vec{r}'\times\vec{r}' = \vec{0}$.

Från l_2: $d\vec{r}' = \frac{d\vec{r}'(\phi)}{d\phi}d\phi = \ldots = (-a\sin\phi\,\hat{z} + a\cos\phi\,\hat{x})\,d\phi$

$d\vec{r}'\times\vec{r}' = \ldots = -a^2\hat{y}\,d\phi$

$\vec{B}(\vec{0}) = \frac{\mu_0}{4\pi} I\hat{y}\int_0^{\pi}\frac{d\phi}{a} = \boxed{\frac{\mu_0 I}{4a}\hat{y}}$

3. $\rho(\vec{r}) = q a^3 e^{-\alpha r}$

Använd Gauss lag. Av symmetri $\vec{E}(\vec{r}) = E(r)\hat{r}$

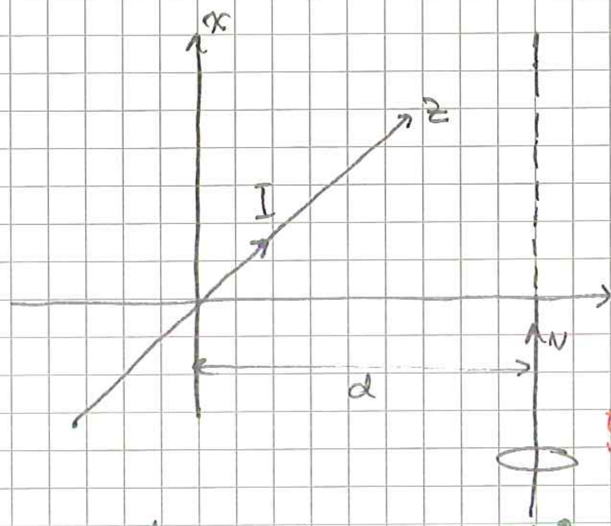
$\underset{r=\text{konst}}{\iint} E\cdot\hat{n}\,dS = \frac{Q_{enc}}{\varepsilon_0}$ | VL: $4\pi r^2 E(r)$

HL: Innesluten laddning i ett klot med radie r

$Q_{enc}(r) = 4\pi\int_0^r \rho(r')r'^2 dr' = 4\pi q(-\alpha^2 r^2 e^{-\alpha r} - 2e^{-\alpha r} + 2)$

$\Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{Q_{enc}(r)\hat{r}}{4\pi r^2\varepsilon_0} = \ldots}$

4.



$\vec{v} = v\hat{x}$ $\quad \vec{r} = (vt, d, 0)$

$d >> a$

$\vec{B}$-fältet från I: $\vec{B}(\vec{r}) = \frac{\hat{\phi}}{2\pi r_c}\mu_0 I$

Flödet genom slingan:

(Välj $\hat{n} = -\hat{x}$)

$\phi(t) = -\pi a^2 \hat{x}\cdot\vec{B}(t)$

$\hat{\phi} = \sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

Avståndet till slingan $r_c(t) = \sqrt{(vt)^2 + d^2}$

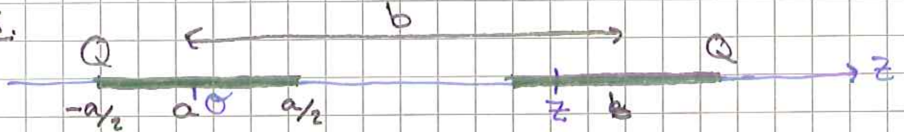
$\hat{x}\cdot\hat{\phi} = -\sin\phi = -\frac{y}{r_c} = -\frac{d}{\sqrt{(vt)^2+d^2}}$! Viktigt att ej glömma! $\hat{x}\cdot\hat{\phi}$!

$$\phi(t) = \pi a^2 \frac{d}{\sqrt{(vt)^2+d^2}}\,\frac{\mu_0 I}{2\pi\sqrt{(vt)^2+d^2}} = \frac{\mu_0 I d a^2}{2((vt)^2+d^2)}$$

vilket ger emk:

$$\boxed{\mathcal{E} = -\frac{d\phi}{dt} = \frac{\mu_0 I a^2 d v^2 t}{((vt)^2+d^2)^2}\ ; \quad P = \frac{\mathcal{E}^2}{R}}$$

5.



Lägg koordinatsystem.

Det elektriska fältet från vänstra staven i en mätpunkt $\vec{r} = z\hat{z}$

källpunkt $\vec{r}' = z'\hat{z}$, $\rho_\ell = \frac{Q}{a}$

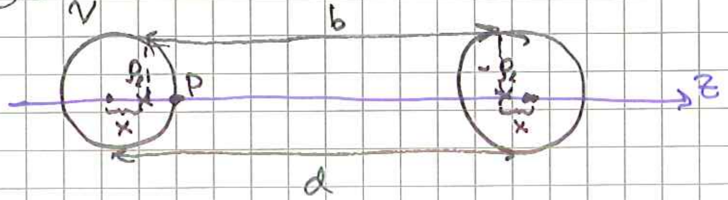
Coulombs lag $\vec{E}(z\hat{z}) = \frac{1}{4\pi\varepsilon_0}\int_{-a/2}^{a/2}\frac{\rho_\ell(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dl' = \frac{\rho_\ell}{4\pi\varepsilon_0}\hat{z}\int_{-a/2}^{a/2}\frac{dz'}{(z-z')^2}$

$$\vec{E}(z\hat{z}) = \frac{Q\hat{z}}{4\pi\varepsilon_0 a}\left(\frac{1}{z-a/2} - \frac{1}{z+\frac{a}{2}}\right).$$

Kraften fås sedan genom integration över högra staven:

$$\boxed{\vec{F} = \rho_\ell\int_{b-\frac{a}{2}}^{b+\frac{a}{2}}\vec{E}(z\hat{z})\,dz = \dots = \frac{Q^2\hat{z}}{4\pi\varepsilon_0 a^2}\ln\frac{b^2}{b^2-a^2}}$$

6. $C = \frac{Q}{V}$



För att få ekvipotentialytor på båda cyl. lägg linjeladdningar ~~täthet~~ i resp. spegelpunkt.

$$\begin{cases} x = \frac{a^2}{b+x} \\ 2x+b = d\end{cases} \Rightarrow \begin{cases} b = \sqrt{d^2-4a^2} \\ x = (d-\sqrt{d^2-4a^2})/2\end{cases}$$

SPEGLING FRÅN F.S.

Potentialen från en linjeladdning $V = -\frac{\rho_\ell}{2\pi\varepsilon_0}\ln(r_c)$. Beräkna potentialen i punkten P.

$$V/2 = -\frac{\rho_\ell}{2\pi\varepsilon_0}\left[\ln(a-x) - \ln(b-a+x)\right] \Rightarrow \boxed{C = \frac{\rho_\ell}{V} = \frac{\pi\varepsilon_0}{\ln\frac{b-a+x}{a-x}} = \dots}$$