12/3-2012 Skrivningsgenomgång Duggan.

(13)

1a) $\begin{cases} u_t' - 2u_{xx}'' = 0 \\ u(0,t) = 0 \\ u(1,t) = 2 \\ u(x,0) = 3 \end{cases}$ Vanligt fel att glömma värmeledningskoefficienten.

2 Lös 1a). $u_{stat} = 2x$. $u = 2x + v$

$V_t' + 2V_{xx}'' = 0$, $u(0,t) = 0$, $V(1,t) = 0$, $V(x,0) = 3 - 2x$

$v = \sum_{k=1} C_k e^{-2k^2\pi^2 t} \sin(k\pi x)$ $(\ldots)$

3. $f(x) = x^2$ $I \in [0,1]$ L_2 normen $\|f\| = \left(\int_0^1 |f(x)|^2 dx\right)^{1/2}$

$\|x^2 - ax - b\|_{L_2} = \left(\int_0^1 (x^2 - ax - b)^2 dx\right)^{1/2}$

$1 \quad \varphi_0 = 1.$

$x \quad \varphi_1 = x - \frac{1}{2} \qquad \varphi_0 \perp \varphi_1$

$P_{[\varphi_0,\varphi_1]} x^2 = c_0\varphi_0 + c_1\varphi_1 = \frac{(\varphi_0|x^2)}{\|\varphi_0\|^2}\varphi_0 + \frac{(\varphi_1|x^2)}{\|\varphi_1\|^2}\varphi_1$

$= x - \frac{1}{6}$

4. $-\frac{d^2}{dx^2} - 2\frac{d}{dx}$ symmetrisk ~~på~~ i $L_2(e^{2x}, [0,1])$

$(f|g) = \int_0^1 f(x)g(x)e^{2x}dx$

$(Au|v) = (u|Av)$. $A = -\frac{1}{e^{2x}}\frac{d}{dx}\left(e^{2x}\frac{d}{dx}u\right)$

$\frac{1}{e^{2x}} e^{2x}(2u' + u'')$

∴ Sturm-Liouville op. $\Rightarrow$ symmetrisk.

$\Rightarrow$ positivt semidefinit $(Au|u) \geq 0$

$\Leftrightarrow$ egenvärden $\lambda \geq 0$

D_A av rätt typ.

Men pos. def? $(Au|u) > 0$ för alla $u \neq 0$.

$\Leftrightarrow$ alla $\lambda > 0$

finns $\lambda = 0$? dvs $Au = 0$!

dvs $-u'' + 2u' = 0$ $\quad u \in D_A$

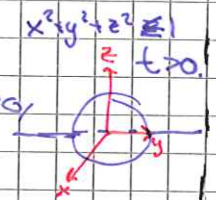
$A + Be^{-2x}$ $\quad u = 1$. (tex). (konst).

$\Rightarrow$ ej pos. def.

6b) Klot i vatten.

$\begin{cases} \partial_t u(x,y,z,t) - a\Delta u(x,y,z,t) = 0 \\ u(x,y,z,t) = 50 \quad x^2+y^2+z^2 = 1 \quad z<0 \quad t>0 \\ u(x,y,z,t) = 20 \quad x^2+y^2+z^2 = 1,\ z>0 \quad t>0 \\ u(x,y,z,0) = \quad x^2+y^2+z^2 < 1 \end{cases}$

$x^2+y^2+z^2 \leq 1$ $\quad t > 0$.



$V(x,y,z) = \lim_{t\to\infty} u(x,y,z,t)$

Enklare $\begin{cases} -\Delta V(x,y,z) = 0 & x^2+y^2+z^2 < 1 \\ V(x,y,z) = 50 & x^2+y^2+z^2 = 1 \quad z>0 \\ \quad = 20 & x^2+y^2+z^2 = 1 \quad z<0 \end{cases}$

Andra koordinatsystem.

$\begin{cases} -\left(\partial_r^2 + \frac{2}{r}\partial_r + \frac{1}{r^2}\partial_s(1-s^2)\partial_s\right)w = 0 & 0<r<1 \\ & -1<s<1 \\ w(1,s) = \begin{cases} 20 & s>0 \\ 50 & s<0 \end{cases} \end{cases}$

$= 35 + 15\,\mathrm{sgn}(s)$

$+ w(r,s)$ begränsad för $r \to 0$

$w(r,s) = \sum_{k=1}^{\infty} c_k R_k(r) P_k(s)$

$\frac{-(R''P + \frac{2}{r}R'P)}{RP} = \frac{\frac{1}{r^2}(\partial_s(1-s^2)\partial_s P)R}{RP}$

$-\frac{(R'' + \frac{2}{r}R')}{R} r^2 = \frac{\partial_s(1-s^2)\partial_s P}{P} = -\mu$

$R'' + \frac{2}{r}R' - \frac{\mu}{r^2}R = 0$

$\partial_s(1-s^2)\partial_s P + \mu P = 0$ Löses av Legendrepolynom.

$\mu = l(l+1)$

Ortogonal bas $L_2(1, [-1,1])$

$\Delta \qquad (f|g) = \iiint f\,g\,dx\,dy\,dz$

$= \iiint f\,g\,r^2 \sin\theta\,dr\,d\theta\,d\varphi$

$= \int_0^1 R_1(r)R_2(r)\,r^2 dr \int_0^{\pi} P_1 P_2 \underbrace{\sin\theta\,d\theta}_{ds}$

$\int_0^{2\pi} \varphi_1(\varphi)\varphi_2(\varphi)\,d\varphi$

$R'' + \frac{1}{R}r' + \left(1 - \frac{\mu^2}{r^2}\right)R = 0$ Bessels differentialekv.

har lösningen

$R(r) = a J_\mu(r) + b Y_\mu(r)$

begr. i 0 $\quad$ obegr. i 0.