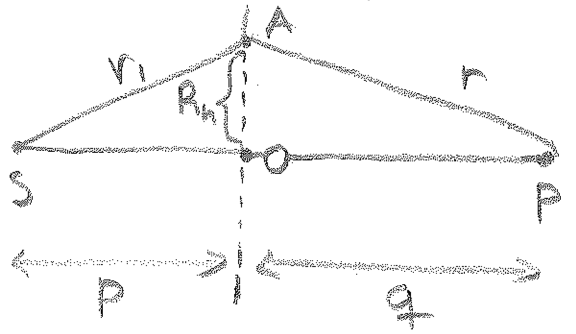
Föreläsning 12 11/05-15

Michelsons interferometer - Amplituddelande
Young's dubbelspalt - Vågfrontsdelande

Uppgift 13.6



Sträckan S-A-P fasskiftas $\frac{n\lambda}{2}$ i förhållande till S-O-P

$r_1 = (R_n^2 + p^2)^{1/2}$

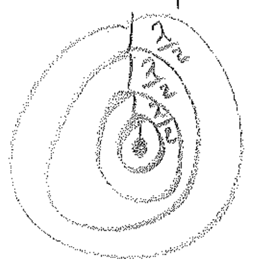
$r = (R_n^2 + q)^{1/2}$

$\Rightarrow (r_1 + r) - (p+q) = \frac{n\lambda}{2}$

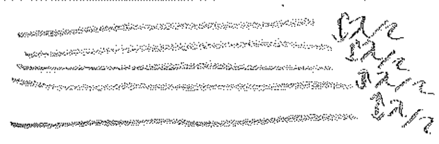
$\Rightarrow \left(\frac{1}{p} + \frac{1}{q}\right) = \frac{n\lambda}{R_n^2}, \quad \frac{1}{q} + \frac{1}{p} = \frac{1}{L}$

$\Rightarrow R_n \simeq \sqrt{nL\lambda}$

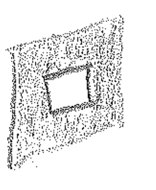
Fresnelplatta



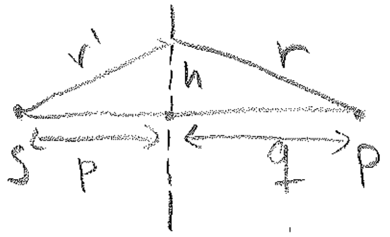
Punktkälla som sänder ut sfäriska vågor.



För spalt



$E_P = C_1 e^{-i\omega t} \iint e^{ik(r-r')} dA$, försummar $F(\theta), \frac{1}{rr'}$



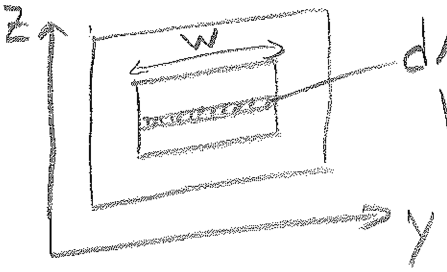
Vad är $r' + r$?

$r' = (p^2 + h^2)^{1/2} = p\left(1 + \frac{h^2}{p^2}\right)^{1/2} \simeq p\left(1 + \frac{h^2}{2p^2}\right)$

$= p + \frac{h^2}{2p}$

$$r + r' \simeq \underbrace{(p+q)}_{\equiv D} + \underbrace{\left(\frac{1}{p}+\frac{1}{q}\right)}_{\equiv \frac{1}{L}} \frac{h^2}{2} \Rightarrow r' + r = D + \frac{h^2}{2L}$$

$$E_p = C_1 e^{-i\omega t} \iint e^{ik\left(D+\frac{h^2}{2L}\right)} \, dA$$



$dA = W\,dz$

$h = z \quad \Rightarrow E_p = C_1 e^{-i\omega t} \int e^{ik\left(D+\frac{z^2}{2L}\right)} w\,dz$

ändras inte

$$E_p = C_1 W e^{i(kD-\omega t)} \int_{z_1}^{z_2} e^{ik\frac{z^2}{2L}} \, dz \quad , \quad \text{Var. byte} \left[\frac{kz^2}{2L} = \frac{\pi z^2}{L\lambda}\right]$$

$$v = z\sqrt{\frac{2}{L\lambda}} \quad , \quad z = v\sqrt{\frac{\lambda L}{2}}$$

$$dz = \sqrt{\frac{\lambda L}{2}}\, dv$$

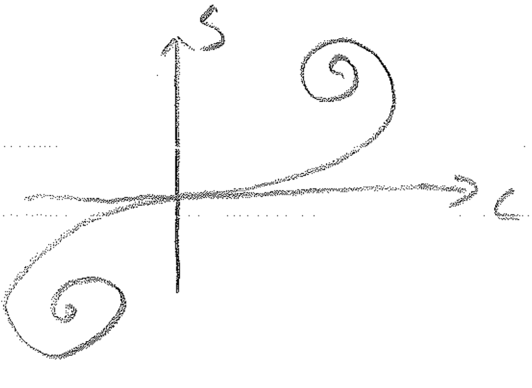
$$\Rightarrow E_p = \underbrace{W\sqrt{\frac{\lambda L}{2}}\, C_1}_{A_p} e^{i(kD-\omega t)} \int_{v_1}^{v_2} e^{i\frac{\pi v^2}{2}} \, dV$$

$$= A_p e^{i(kD-\omega t)} \left[\int_{v_1}^{v_2} \cos\frac{\pi v^2}{2}\, dv + i\int_{v_1}^{v_2} \sin\frac{\pi v^2}{2}\, dv\right]$$

Fresnel integralerna

$$C(v) = \int_0^v \cos\left(\frac{\pi v^2}{2}\right) dv$$

$$S(v) = \int_0^v \sin\left(\frac{\pi v^2}{2}\right) dv$$



Cornuspiralen är ett sätt att lösa integ. grafiskt.

Integ löser man numeriskt.

Amplitud $E_p = A_p e^{i(kD-\omega t)} \left[(C(v_2) - C(v_1)) + i(S(v_2) - S(v_1)) \right]$

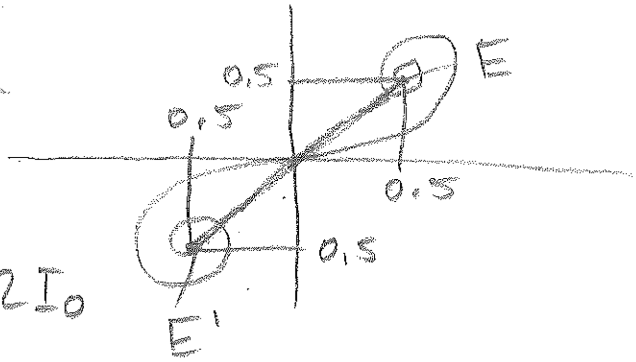
$I_0 = \frac{1}{2} \varepsilon_0 c |E_p|^2$

$= \frac{1}{2} \varepsilon_0 c |A_p|^2 \underbrace{\left[(C(v_2) - C(v_1))^2 + (S(v_2) - S(v_1))^2 \right]}_{\text{Längd på vektor i Cornu-spiralen från } v_1 \to v_2}$

Ex 1. Inget hinder $z -\infty \Rightarrow \infty$, $v -\infty \Rightarrow \infty$

$C(\infty) = 0,5$

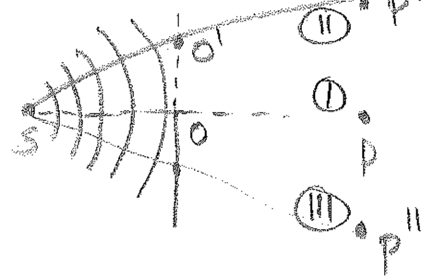
$S(\infty) = 0,5$



$I_v = I_0 (E'E)^2 =$

$= I_0 [1^2 + 1^2] = 2 I_0$

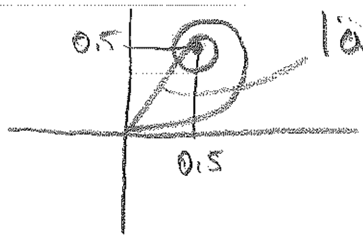
Ex 2. Rak kant



I varje pkt man räknar måste man def om sitt syst.

(I) S-O-P kanten av skuggan

Alla vågor $z = 0 \Rightarrow +\infty$ ska adderas



längden $= \frac{1}{\sqrt{2}}$

$I_p = I_0 (\overline{OE}) = \frac{1}{2} I_0 = \frac{1}{4} I_u$

(II) S-O'-P' över kanten = fler vågor än (I)

Nu är O' centrum, z lite neg $\Rightarrow \infty$

(III) S-O''-P''

z lite pos $\Rightarrow \infty$