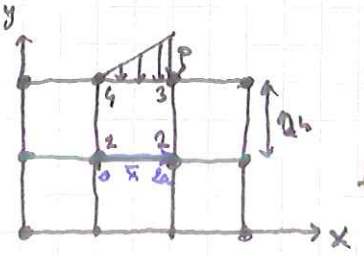
Extentagenomgång

22/5 2013.

Problem 4.



$$f_b = \int_L t N^{eT} \mathbf{t} \, dL^e$$

$$\mathbf{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} 0 \\ -p\frac{\bar{x}}{2a} \end{bmatrix}$$

a) $N_3(\bar{x}) = \frac{\bar{x}}{2a}$

$N_4(\bar{x}) = 1 - \frac{\bar{x}}{2a}$

$$f_b = \int_{L^e} t \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \\ N_4 & 0 \\ 0 & N_4 \end{bmatrix} \begin{bmatrix} 0 \\ t_y \end{bmatrix} dL = t \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \int_0^{2a} N_3 t_y \, d\bar{x} \\ 0 \\ \int_0^{2a} N_4 t_y \, d\bar{x} \end{bmatrix}$$

$$t\int_0^{2a} N_3 t_y \, d\bar{x} = t\int_0^{2a} \frac{\bar{x}}{2a}\left(-p\frac{\bar{x}}{2a}\right) d\bar{x} = -p\frac{t}{4a^2}\left[\frac{\bar{x}^3}{3}\right]_0^{2a} = -\frac{pta \cdot 2}{3}.$$

$$t\int_0^{2a} N_4 t_y \, d\bar{x} = t\int_0^{a} \left(1 - \frac{\bar{x}}{2a}\right)\left(-p\frac{\bar{x}}{2a}\right) d\bar{x} = -\frac{1}{3}pat.$$

b) Isoparametrisk avbildning

$\xi(\bar{x}) = k\bar{x} + m.$ $\begin{cases} \xi(0) = -1 \Rightarrow \boxed{m = -1} \\ \xi(2a) = 1 = 2ak - 1 \Rightarrow \boxed{k = \frac{1}{a}} \end{cases} \Rightarrow \boxed{\xi = \frac{\bar{x}}{a} - 1}$

$d\bar{x} = a\,d\xi$ $\quad N_3(\xi) = \frac{1}{2}(1+\xi)$ $\quad w_1 = 1$ $\quad \xi_1 = -\frac{1}{\sqrt{3}}$ $\quad t_y(\bar{x}) = -p\frac{\bar{x}}{2a} \Rightarrow t_y(\xi) = -\frac{p}{2a}(a\xi + a)$

$N_4(\xi) = \frac{1}{2}(1-\xi)$ $\quad w_2 = 1$ $\quad \xi_2 = \frac{1}{\sqrt{3}}$

$$t\int_0^{2a} N_3 t_y \, d\bar{x} = t\int_{-1}^{1} \frac{1}{2}(1+\xi)\left(-\frac{p}{2}(1+\xi)a\right)d\xi = -\frac{pt}{4}a\int_{-1}^{1}(1+\xi)^2 d\xi \approx \cancel{-\frac{pta}{4}\left(1(1-\frac{1}{\sqrt{3}})^2\right.}$$

$$\approx -\frac{pta}{4}\left(1\cdot\left(1-\frac{1}{\sqrt{3}}\right)^2 + 1\cdot\left(1+\frac{1}{\sqrt{3}}\right)^2\right) = -\frac{pta}{4}\left(1 + \frac{1}{3} + 1 + \frac{1}{3}\right) = -\frac{2pta}{3}$$

$$\int_0^{2a} N_4 t_y \, d\bar{x} = t\int_{-1}^{1} \frac{1}{2}(1-\xi)\left(-\frac{p}{2}(\xi+1)a\right) d\xi = -\frac{pta}{4}\int_{-1}^{1}(1-\xi)(1+\xi)\,d\xi =$$

$$= -\frac{pta}{4}\left(1\cdot\left(1-\left(-\frac{1}{\sqrt{3}}\right)\right)\left(1-\frac{1}{\sqrt{3}}\right) + 1\cdot\left(1-\frac{1}{\sqrt{3}}\right)\left(1+\frac{1}{\sqrt{3}}\right)\right) = -\frac{1}{3}pat$$

Problem 5. $\cdot$ $\operatorname{div}(\nabla p) = 0$

$$\int_A w \operatorname{div}(\nabla p)\, dA = 0$$

$$\oint_L w (\nabla p)^T n\, dL - \int_A (\nabla w)^T \cdot \nabla p\, dA = 0. \quad \text{weak form.}$$

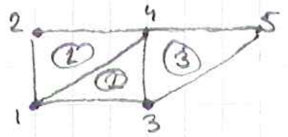
$$p = \mathbb{N} \mathbb{a} \qquad \nabla p = \nabla \mathbb{N} \mathbb{a} = \mathbb{B} \mathbb{a}.$$

Galerkin. $w = \mathbb{N} \mathbb{c} = \mathbb{c}^T \mathbb{N}^T \qquad \nabla w = \mathbb{B} \mathbb{c}$

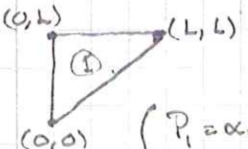
$$\oint_L \mathbb{c}^T \mathbb{N}^T (\nabla p)^T n\, dL - \mathbb{c}^T \int \mathbb{B}^T \mathbb{B}\, dA\, \mathbb{a} = 0.$$

$$\mathbb{c}^T \left(\oint \mathbb{N}^T (\nabla p)^T n\, dL - \int \mathbb{B}^T \mathbb{B}\, dA\, \mathbb{a} \right) = 0.$$

$$\underbrace{\oint_L \mathbb{N}^T (\nabla p)^T n\, dL}_{f_b} - \underbrace{\int_A \mathbb{B}\mathbb{B}\, dA}_{\mathbb{K}}\, \underbrace{\mathbb{a}}_{\mathbb{a}} = 0.$$



Edof: $\begin{pmatrix} 1 & 1 & 4 & 2 \\ 2 & 1 & 3 & 4 \\ 3 & 3 & 5 & 4 \end{pmatrix}$



C-matrismetoden. Element 1 & 3.

$$p = \alpha_1 + \alpha_2 x + \alpha_3 y = \underbrace{(1 \;\; x \;\; y)}_{\bar{\mathbb{N}}^e} \underbrace{\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}}_{\alpha}$$

$$\begin{cases} p_1 = \alpha_1 \\ p_2 = \alpha_1 + \alpha_2 L + \alpha_3 L \\ p_3 = \alpha_1 + \alpha_3 L \end{cases} \Leftrightarrow \underbrace{\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}}_{\mathbb{a}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & L & L \\ 1 & 0 & L \end{bmatrix}}_{\mathbb{C}} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}}_{\alpha} \Rightarrow \alpha = \mathbb{C}^{-1} \mathbb{a}.$$

$$p = \bar{\mathbb{N}}^e \mathbb{C}^{-1} \mathbb{a} = \mathbb{N}^e \mathbb{a}$$

$$\nabla p = \mathbb{B} \mathbb{a}$$

$$\nabla \mathbb{N}^e = \nabla(\bar{\mathbb{N}}^e \mathbb{C}^{-1}) = \nabla((1 \;\; x \;\; y)\mathbb{C}^{-1}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbb{C}^{-1} = \mathbb{B}$$

$$\mathbb{K}_1 = \int_{A_1} \mathbb{B}^T \mathbb{B}\, dA_1 = \int_{A_1} (\mathbb{C}^{-1})^T \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbb{C}^{-1}\, dA = (\mathbb{C}^{-1})^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbb{C}^{-1} \int_{A_1} dA_1$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix} = \mathbb{K}_1 = \mathbb{K}_3. \qquad \mathbb{K}_2 = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \mathbb{K}_2^e.$$

Liknande for $\mathbb{K}_2$.

mha Edof'a

$$\mathbb{K} = \frac{1}{2} \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 3 & -2 & 0 \\ 0 & -1 & -2 & 4 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \Rightarrow \frac{1}{2} \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 3 & -2 & 0 \\ 0 & -1 & -2 & 4 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ 0 \\ p_3 \\ 0 \\ 0 \end{bmatrix} = f$$

$$= \begin{bmatrix} \int_{L_{12}} N_1 \cdot (-\rho a)\, dL_{12} + \\ \int_{L_{12}} N_2 (-\rho a)\, dL_{12} \quad + \int_{2-4} N_2 (\nabla p)^T n\, dL_{2-4} \\ 0 \\ \vdots \text{ endast intresserade av } P_1, P_3 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} P_1 \\ P_3 \end{bmatrix} = \begin{bmatrix} \int_{L_{1-2}} N_1 (-\rho a)\, dL_{1-2} \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{\rho a L}{2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_3 \end{bmatrix} = \frac{2}{5}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -\frac{\rho a L}{2} \\ 0 \end{bmatrix}$$

Uppgift x.

$$\operatorname{div}\left(h_0^3 \nabla p\right) = 6 \cdot -\frac{2}{3} = -4.$$

$$\int_A B^T h_0^3 B\, dA \cdot a = \oint N^T \left(h_0^3 (\nabla p) n\right) dL + 4 \int N\, dA.$$