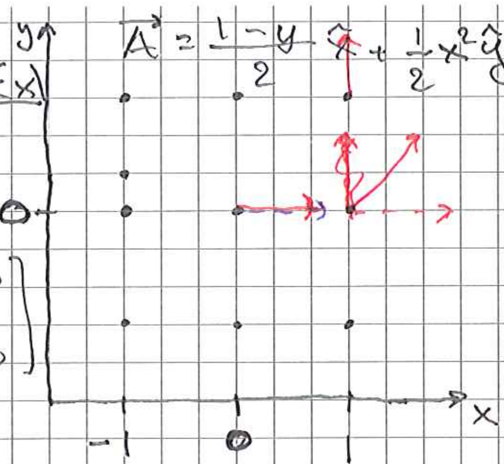
16/1-2012 (1)

Ex $\vec{A} = \frac{1-y}{2}\hat{x} + \frac{1}{2}x^2\hat{y}$



$\hat{x} = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad \hat{y}\begin{bmatrix}0\\1\\0\end{bmatrix}$

$\vec{V} = \begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix} \quad a(x,y,z)$

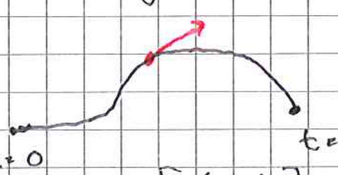
$\vec{U}\cdot\vec{V} = U_xV_x + U_yV_y + U_zV_z.$

$\vec{U}\times\vec{V} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ U_x & U_y & U_z\\ V_x & V_y & V_z\end{vmatrix} = (U_yV_z - U_zV_y)\hat{x} + (U_xV_z - U_zV_x)\hat{y} + (U_xV_y - U_yV_y)$

$f(x,y) = C$

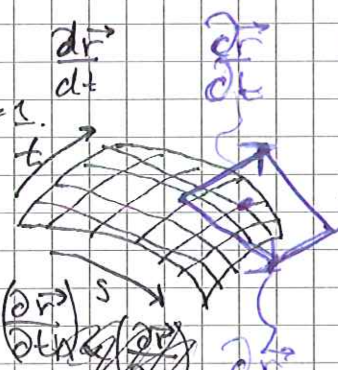
$\vec{g}\cdot\hat{n}$ - (riktningsderivata)

Rymdkurva $\vec{r}(t) = \begin{bmatrix}x(t)\\y(t)\\z(t)\end{bmatrix}$



$\frac{d\vec{r}}{dt}$

$\vec{r}(s,t) = \begin{bmatrix}x(s,t)\\y(s,t)\\z(s,t)\end{bmatrix}$



$\frac{\partial\vec{r}}{\partial t}$

$\hat{n} \propto \left(\frac{\partial\vec{r}}{\partial s}\right)\times\left(\frac{\partial\vec{r}}{\partial t}\right)$

$\frac{\partial\vec{r}}{\partial s}$

1 dimension.

$f(x): \quad \frac{df}{dx} \quad \frac{d}{dx}$

$\nabla = \begin{bmatrix}\partial/\partial x\\ \partial/\partial y\\ \partial/\partial z\end{bmatrix} \quad \text{grad} f = \nabla f = \begin{bmatrix}\frac{\partial f}{\partial x}\\ \frac{\partial f}{\partial y}\\ \frac{\partial f}{\partial z}\end{bmatrix}$

$\underset{\text{vektorfält}}{\nabla\cdot\vec{V}} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ } Divergens

Fysikalisk tolkning av divergens:
Hur mycket som produceras i ett område.

$\nabla\times\vec{V} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ V_x & V_y & V_z\end{vmatrix} = \begin{bmatrix}\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\\ \frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\\ \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\end{bmatrix}$ Rotation rot V

Rotationsvektorn

$\nabla\cdot(\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$\Delta = \nabla^2 = \nabla\cdot\nabla$

$\nabla\times(\nabla f) = \begin{vmatrix}\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial y}\right)\\ \vdots\\ \vdots\end{vmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

liknande räkningar.

$\nabla\cdot(\nabla\times\vec{V}) = 0$

Ex Visa: $\nabla\times(f\vec{C}) = f(\nabla\times\vec{C}) - \vec{C}\times(\nabla f)$
gäller om $f = x^2$ och $\vec{C} = 3\hat{y} + y\hat{z}$.

VL: $\nabla\times(f\vec{C}) = \nabla\times\begin{bmatrix}fC_x\\fC_y\\fC_z\end{bmatrix} = \nabla\times\begin{bmatrix}0\\3x^2\\x^2y\end{bmatrix} =$

$= \begin{bmatrix}x^2 - 0\\ 0 - 2xy\\ 6x - 0\end{bmatrix} = \begin{bmatrix}x^2\\ -2xy\\ 6x\end{bmatrix}$

HL: $f(\nabla\times\vec{C}) - \vec{C}\times(\nabla f) = x^2\left(\nabla\times\begin{bmatrix}0\\3\\y\end{bmatrix}\right) -$

$- \begin{bmatrix}0\\3\\y\end{bmatrix}\times\begin{bmatrix}2x\\0\\0\end{bmatrix} = x^2\begin{bmatrix}1-0\\0-0\\0-0\end{bmatrix} - \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 3 & y\\ 2x & 0 & 0\end{vmatrix} =$

$= \begin{bmatrix}x^2\\0\\0\end{bmatrix} - \begin{bmatrix}0\\2xy\\0-6x\end{bmatrix} = \begin{bmatrix}x^2\\-2xy\\6x\end{bmatrix}$ Vad som skulle visas.