Kont.sys.: Studie av partiella differential-ekvationer. (PDE)

partiella – flera variabler. $x, y, \ldots$
tiden, $t, x, y \ldots$

ordinära – 1 variabel.

Repetition – ordinära:

Ex: $u' = 0$; $u' = xu + 3$, $u' = u^2 x^2$, $e^{u'u} = -2$
~~$u'u''$~~ $u'' + 2u' + 7u = 0$ . .

Viktiga frågor: 1. Finns det en lösning? (Existens).
2. Finns det bara en eller många lösningar? (Entydighet)
3. Beror lösningen på ekvationen (kontinuerligt)

$u' = 0 \Leftrightarrow u = C$. För entydighet $\Rightarrow$ Begynnelsevärde.

Sats $\begin{cases} u'(x) = f(x, u(x)), & f \text{ deriverbar} \\ u(0) = u_0 \end{cases}$
har precis en lösning

Ex $u' = xu + 3x \Leftrightarrow u' - xu = 3x$
$[g' = -x]$. $g = \int_0 -x\,dx = -\frac{x^2}{2}$

$e^{-\frac{x^2}{2}}(u' - xu) = 3e^{-\frac{x^2}{2}}x$

$\left(u e^{-\frac{x^2}{2}}\right)' = 3e^{-\frac{x^2}{2}}x$

$u e^{-\frac{x^2}{2}} = -3e^{-\frac{x^2}{2}} + C$

$u(x) = Ce^{\frac{x^2}{2}} - 3$

$u(0) = 1 \Rightarrow C = 4$

Ex $\begin{cases} u' = u^2 x^2 \\ u(0) = 1 \end{cases}$

Seperera variabler

$\frac{u'}{u^2} = x^2 \Rightarrow \left(-\frac{1}{u}\right)' = \left(\frac{x^3}{3} + C\right)'$

$-\frac{1}{u} = \frac{x^3}{3} + C \; \ldots$

$u(x) = \frac{3}{3 - x^3}$

$\partial_t u + 2x\,\partial_x u = 0$

$u'_t + 2x\,u'_x = 0$ $\qquad s = \ldots$, $y = \ldots$

$\frac{\partial u}{\partial t} + 2x\frac{\partial u}{\partial x} = 0$

$\partial_s u = 0$
$u(s, y) = C(y)$.

• Kända PDE

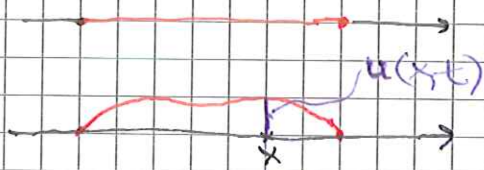
Laplace $\Delta u = 0$ $\quad (\Delta = \partial_x^2 + \partial_y^2)$
$(\partial_x^2 + \partial_y^2 + \partial_z^2)$

Poisson $\Delta u = f$

Värmeledning
Diffusion $\partial_t u - c\Delta u = 0$

Vågekvation $\partial_t^2 u - c^2 \Delta u = 0$

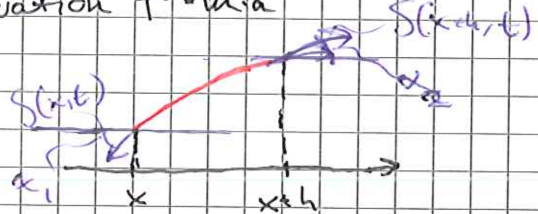
Svängande sträng



Spännkraft. $S(x,t)$ är tangentiell
$S = |S(x,t)|$ konstant.

Yttre kraft $f(x,t)\,\hat{e}_y$ $\quad$ | ρ_ℓ längddensitet

Kraftekvation $F = m \cdot a$



$h\rho_\ell \frac{\partial^2 u}{\partial t^2} \approx S \cdot \sin\alpha_2 - S \cdot \sin\alpha_1 + fh$

$\sin\alpha_1 \approx \alpha_1 \approx \tan\alpha_1 = \frac{\partial u}{\partial x}(x, t)$

$\sin\alpha_2 \quad \frac{\partial u}{\partial x}(x+h, t)$

$\rho_\ell \frac{\partial^2 u}{\partial t^2} \approx S \frac{\frac{\partial u}{\partial x}(x+h,t) - \frac{\partial u}{\partial x}(x,t)}{h} + f$

$h \to 0 \Rightarrow \boxed{\rho_\ell \frac{\partial^2 u}{\partial t^2} = S\frac{\partial^2 u}{\partial x^2} + f}$

$\boxed{\frac{\partial^2 u}{\partial t^2} - \frac{S}{\rho_\ell}\frac{\partial^2 u}{\partial x^2} = \frac{f}{\rho_\ell}}$ Vågekvationen