

# Lösningar kapitel 2

Endimensionell analys

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**2.1****a)**

$$(x+3)(x-3) - (x+3)^2 = x^2 - 3^2 - (x^2 + 2x \cdot 3 + 3^2) = x^2 - 9 - (x^2 + 6x + 9) \\ = x^2 - 9 - x^2 - 6x - 9 = -6x - 18$$

**c)**

$$(3x+5)^2 - (3x-5)^2 = (3x)^2 + 2 \cdot 3x \cdot 5 + 5^2 - (3x-5)^2 = 9x^2 + 30x + 25 - (3x+5)^2 \\ = 9x^2 + 30x + 25 - ((3x)^2 + 2 \cdot 3x \cdot (-5) + (-5)^2) \\ = 9x^2 + 30x + 25 - (9x^2 - 30x + 25) = 60x$$

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**2.2**

$$(a-b)^3 = (a-b)^2(a-b) = (a^2 - 2ab + b^2)(a-b) = a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3 \\ = a^3 - 3a^2b + 3ab^2 - b^3$$

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**2.3**

$$(a-b)(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16}) \\ = (a^2-b^2)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16}) \\ = (a^4-b^4)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16}) \\ = (a^8-b^8)(a^8+b^8)(a^{16}+b^{16}) \\ = (a^{16}-b^{16})(a^{16}+b^{16}) \\ = a^{32} - b^{32}$$

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**2.5****a)**

$$\frac{1}{7} - \left(\frac{15}{14} + \frac{1}{2}\right) = \frac{1}{7} - \left(\frac{15}{14} + \frac{7}{14}\right) = \frac{1}{7} - \frac{22}{14} = \frac{1}{7} - \frac{11}{7} = -\frac{10}{7}$$

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**2.6****a)**

$$\frac{1}{60} + \frac{1}{108} - \frac{1}{72} = \frac{108}{6480} + \frac{60}{6480} - \frac{1}{72} = \frac{168}{6480} - \frac{1}{72} = \frac{168}{6480} - \frac{90}{6480} = \frac{78}{6480} = \frac{13}{1080}$$

**b)**

$$\frac{3}{4} - \frac{5}{6} + \frac{1}{9} = \frac{18}{24} - \frac{20}{24} + \frac{1}{9} = -\frac{2}{24} + \frac{1}{9} = -\frac{18}{216} + \frac{24}{216} = \frac{6}{216} = \frac{1}{36}$$

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## 2.7

a)

$$\frac{\frac{a}{2}}{\frac{a}{4}} = \frac{a}{2} \cdot \frac{4}{a} = 2$$

c)

$$\frac{\frac{14a}{a+2}}{\frac{7}{6a+12}} = \frac{14a}{a+2} \cdot \frac{6a+12}{7} = \frac{2a(6a+12)}{a+2} = \frac{12a(a+2)}{a+2} = 12a$$

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## 2.8

a)

$$\begin{aligned} \frac{\frac{3}{5x} - \frac{x}{15}}{\frac{1}{x} - \frac{1}{3}} &= \frac{\frac{45}{75x} - \frac{5x^2}{75x}}{\frac{3}{3x} - \frac{x}{3x}} = \frac{\frac{45-5x^2}{75x}}{\frac{3-x}{3x}} = \frac{3x(45-5x^2)}{75x(3-x)} = \frac{45-5x^2}{25(3-x)} = \frac{5(9-x^2)}{25(3-x)} \\ &= \frac{9-x^2}{5(3-x)} = \frac{(3+x)(3-x)}{5(3-x)} = \frac{x+3}{5} \end{aligned}$$

b)

$$\frac{x^2+1}{1+\frac{1}{x^2}} = \frac{x^2+1}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \frac{x^2+1}{\frac{x^2+1}{x^2}} = \frac{x^2(x^2+1)}{x^2+1} = x^2$$

c)

$$\begin{aligned} \frac{\frac{1}{x} - \frac{1}{y}}{\frac{x^2-y^2}{(xy)^2}} &= \frac{\frac{-(x+y)}{xy}}{\frac{x^2-y^2}{(xy)^2}} = -\frac{(xy)^2(x+y)}{xy(x^2-y^2)} = -\frac{xy(x+y)}{x^2-y^2} \\ &= -\frac{xy(x+y)}{(x+y)(x-y)} = -\frac{xy}{x-y} \end{aligned}$$

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## 2.9

a)

$$\begin{aligned} \frac{\frac{x}{y} - \frac{y}{x}}{\frac{x}{y} + \frac{y}{x} - 2} &= \frac{\frac{x^2-y^2}{xy}}{\frac{x^2+y^2}{xy} - 2} = \frac{\frac{x^2-y^2}{xy}}{\frac{x^2-2xy+y^2}{xy}} = \frac{\frac{x^2-y^2}{xy}}{\frac{(x-y)^2}{xy}} = \frac{xy(x^2-y^2)}{xy(x-y)^2} = \frac{x^2-y^2}{(x-y)^2} \\ &= \frac{(x+y)(x-y)}{(x-y)^2} = \frac{x+y}{x-y} \end{aligned}$$

b)

$$\begin{aligned}\frac{\frac{16x^4}{81} - y^4}{\frac{2x}{3} + y} &= \frac{\frac{16x^4 - 81y^4}{81}}{\frac{2x+3y}{3}} = \frac{3(16x^4 - 81y^4)}{81(2x+3y)} = \frac{16x^4 - 81y^4}{27(2x+3y)} \\ &= \frac{(4x^2 + 9y^2)(4x^2 - 9y^2)}{27(2x+3y)} = \frac{(4x^2 + 9y^2)(2x+3y)(2x-3y)}{27(2x+3y)} \\ &= \frac{(4x^2 + 9y^2)(2x-3y)}{27} = \frac{8x^3 - 12x^2y + 18xy^2 - 27y^3}{27}\end{aligned}$$

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## 2.10

a)

$$\begin{aligned}\frac{1}{R} &= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{3}{6} + \frac{2}{6} + \frac{1}{4} = \frac{5}{6} + \frac{1}{4} = \frac{20}{24} + \frac{6}{24} = \frac{26}{24} = \frac{13}{12} \\ &\iff R = \frac{12}{13}\end{aligned}$$

b)

$$\frac{1}{3} = \frac{1}{R_1} + \frac{1}{5} \iff \frac{1}{R_1} = \frac{1}{3} - \frac{1}{5} \iff \frac{1}{R_1} = \frac{5}{15} - \frac{3}{15} \iff \frac{1}{R_1} = \frac{2}{15} \iff R_1 = \frac{15}{2}$$

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## 2.11

$$\begin{aligned}\frac{1}{600} + \frac{1}{b} &= \frac{1}{100} \iff \frac{1}{b} = \frac{1}{100} - \frac{1}{600} \iff \frac{1}{b} = \frac{6}{600} - \frac{1}{600} \\ &\iff \frac{1}{b} = \frac{5}{600} \iff \frac{1}{b} = \frac{1}{120} \iff b = 120\end{aligned}$$

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## 2.13

$$\frac{3 + \sqrt{5}}{2 + \sqrt{5}} = \frac{(3 + \sqrt{5})(2 - \sqrt{5})}{(2 + \sqrt{5})(2 - \sqrt{5})} = \frac{6 - 3\sqrt{5} + 2\sqrt{5} - 5}{2^2 - (\sqrt{5})^2} = \frac{1 - \sqrt{5}}{4 - 5} = \sqrt{5} - 1$$

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## 2.14

a)

$$\frac{1 + 2\sqrt{2}}{3 - \sqrt{2}} = \frac{(1 + 2\sqrt{2})(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \frac{3 + \sqrt{2} + 6\sqrt{2} + 4}{9 - 2} = \frac{7 + 7\sqrt{2}}{7} = 1 + \sqrt{2}$$

c)

$$\frac{2}{\sqrt{x+1} + \sqrt{x-1}} = \frac{2(\sqrt{x+1} - \sqrt{x-1})}{(\sqrt{x+1} + \sqrt{x-1})(\sqrt{x+1} - \sqrt{x-1})}$$

observera konjugatregeln i nämnaren

$$= \frac{2(\sqrt{x+1} - \sqrt{x-1})}{x+1 - (x-1)} = \frac{2(\sqrt{x+1} - \sqrt{x-1})}{2} = \sqrt{x+1} - \sqrt{x-1}$$

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## 2.15

a)

$$\sqrt{12} - \sqrt{3} = \sqrt{4 \cdot 3} - \sqrt{3} = \sqrt{4}\sqrt{3} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}(2 - 1) = \sqrt{3}$$

b)

$$\frac{\sqrt{42}}{\sqrt{6}} = \frac{\sqrt{6 \cdot 7}}{\sqrt{6}} = \frac{\sqrt{6}\sqrt{7}}{\sqrt{6}} = \sqrt{7}$$

c)

$$\sqrt{3}\sqrt{12} = \sqrt{3}\sqrt{3 \cdot 4} = \sqrt{3}\sqrt{3}\sqrt{4} = 3\sqrt{4} = 3 \cdot 2 = 6$$

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## 2.16

a)

$$\frac{\sqrt{162} + \sqrt{98}}{\sqrt{50} + \sqrt{2}} = \frac{\sqrt{81 \cdot 2} + \sqrt{49 \cdot 2}}{\sqrt{25 \cdot 2} + \sqrt{2}} = \frac{9\sqrt{2} + 7\sqrt{2}}{5\sqrt{2} + \sqrt{2}} = \frac{\sqrt{2}(9 + 7)}{\sqrt{2}(5 + 1)} = \frac{16}{6} = \frac{8}{3}$$

b)

$$\frac{\sqrt{(-4)^2}}{\sqrt{4^2}} = \frac{\sqrt{16}}{\sqrt{16}} = 1$$

c)

$$\begin{aligned} \left(\sqrt{12} - \frac{1}{\sqrt{3}}\right)^2 &= (\sqrt{12})^2 - 2\frac{\sqrt{12}}{\sqrt{3}} + \left(\frac{1}{\sqrt{3}}\right)^2 = 12 - 2\frac{\sqrt{3}\sqrt{4}}{\sqrt{3}} + \frac{1}{3} = 12 - 4 + \frac{1}{3} \\ &= 8 + \frac{1}{3} = \frac{24}{3} + \frac{1}{3} = \frac{25}{3} \end{aligned}$$

d)

Använd konjugatregeln

$$\begin{aligned} ((\sqrt{x} + \sqrt{y}) + \sqrt{x+y})((\sqrt{x} + \sqrt{y}) - \sqrt{x+y}) &= (\sqrt{x} + \sqrt{y})^2 - \sqrt{x+y}^2 \\ &= x + 2\sqrt{x}\sqrt{y} + y - (x + y) = 2\sqrt{x}\sqrt{y} = 2\sqrt{xy} \end{aligned}$$

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## 2.18

b)

$$2^7 \cdot 2^{-3} = 2^{7+(-3)} = 2^4$$

d)

$$\frac{3^7}{3^3} = 3^{7-3} = 3^4$$

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**2.19**

a)

$$3^5 \cdot 10^5 \cdot 3^{-3} \cdot 10^3 = 3^{5+(-3)} \cdot 10^{5+3} = 3^2 \cdot 10^8 = 9 \cdot 10^8$$

b)

$$\frac{2^8 \cdot 5^6}{2^6 \cdot 5^5} = \frac{2^8}{2^6} \cdot \frac{5^6}{5^5} = 2^{8-6} \cdot 5^{6-5} = 2^2 \cdot 5 = 4 \cdot 5 = 20$$


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**2.20**

b)

$$(a^3)^{-0.5} \cdot (a^{-5})^{-0.3} = a^{3 \cdot (-0.5)} \cdot a^{-5 \cdot (-0.3)} = a^{-1.5} \cdot a^{1.5} = a^{-1.5+1.5} = a^0 = 1$$

d)

$$\frac{x \cdot x^{-1.6} \cdot x^{0.2}}{x^{-1.4}} = \frac{x^{1+(-1.6)+0.2}}{x^{-1.4}} = \frac{x^{-0.4}}{x^{-1.4}} = x^{-0.4-(-1.4)} = x^1 = x$$


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**2.21**

a)

$$\frac{3^2 \cdot 2^4}{6^3} = \frac{3^2 \cdot 2^4}{(3 \cdot 2)^3} = \frac{3^2 \cdot 2^4}{3^3 \cdot 2^3} = \frac{2}{3}$$

b)

$$\left(\frac{1}{4}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{1}{4}\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{\sqrt{1}}{\sqrt{4}}} = \frac{1}{\frac{1}{2}} = 2$$

c)

$$(\sqrt{64})^{\frac{2}{3}} = 8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$$

d)

$$\left(\frac{1}{3}\right)^{-1} = \frac{1}{\frac{1}{3}} = 3$$

e)

$$2^{(2^3)} = 2^8 = 256$$

f)

$$(2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$$


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## 2.23

a)

$$\frac{a^{3 \cdot 3} a^{-2 \cdot 1}}{a^{0.8}} = \frac{a^{3 \cdot 3 - 2 \cdot 1}}{a^{0.8}} = \frac{a^{1.2}}{a^{0.8}} = a^{1.2 - 0.8} = a^{0.4}$$

b)

$$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = \frac{aa^{\frac{1}{2}}}{(a^2)^{\frac{1}{3}}} = \frac{a^{1+\frac{1}{2}}}{a^{\frac{2}{3}}} = \frac{a^{\frac{3}{2}}}{a^{\frac{2}{3}}} = a^{\frac{3}{2}-\frac{2}{3}} = a^{\frac{5}{6}}$$

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## 2.24

$$\begin{aligned} \frac{(2^6 \cdot 5^5)^3 \cdot 2^{18}}{(2^4 \cdot 5^2)^4 \cdot 5^7} &= \frac{2^{18} \cdot 5^{15} \cdot 2^{18}}{2^{16} \cdot 5^8 \cdot 5^7} = \frac{2^{36} \cdot 5^{15}}{2^{16} \cdot 5^{15}} \\ &= \frac{2^{36}}{2^{16}} = 2^{20} = (2^4)^5 = (2^2 \cdot 2^2)^5 = (4^2)^5 \end{aligned}$$

jämförelse med VL ger att  $x = 2$ .

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## 2.25

a)

Använd konjugatregeln

$$x^2 - 1 = (x + 1)(x - 1)$$

c)

Använd konjugatregeln

$$x^3 - 4x = x(x^2 - 4) = x(x + 2)(x - 2)$$

d)

Använd kvadreringsregeln

$$x^2 - 2x + 1 = (x - 1)^2$$

h)

Bryt ut och använd kvadreringsregeln

$$a^2b + 2ab^2 + b^3 = b(a^2 + 2ab + b^2) = b(a + b)^2$$

i)

Bryt ut  $ab$  och använd kvadreringsregeln

$$a^3b - 2a^2b^2 + ab^3 = ab(a^2 - 2ab + b^2) = ab(a - b)^2$$

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## 2.27

a)

$$x^2 - 7x + xy - 7y = x(x - 7) + y(x - 7) = (x - 7)(x + y)$$

c)

Använd konjugatregeln i sista steget

$$\begin{aligned}x^2y + 2x^2 - y - 2 &= x^2(y + 2) - y - 2 = x^2(y + 2) - (y + 2) \\ &= (y + 2)(x^2 - 1) = (y + 2)(x + 1)(x - 1)\end{aligned}$$

f)

Börja med konjugatregeln i första steget. Använd sen kvadreringsregeln på de två termerna i sista steget

$$(x^2 + y^2)^2 - (2xy)^2 = (x^2 + y^2 + 2xy)(x^2 + y^2 - 2xy) = (x + y)^2(x - y)^2$$

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## 2.28

a)

$$x^2 + 6x + 7 = x^2 + 6x + 9 - 9 + 7 = (x + 3)^2 - 2$$

b)

$$\begin{aligned}x^2 - 7x + 13 &= x^2 - 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 13 = \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 13 \\ &= \left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + 13 = \left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + \frac{52}{4} = \left(x - \frac{7}{2}\right)^2 + \frac{3}{4}\end{aligned}$$

d)

$$x^2 + 5x = x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4}$$

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## 2.29

Börja med att utveckla högerledet

$$(x + b)^2 + c = x^2 + 2bx + b^2 + c$$

Jämför nu med vänsterledet. Vi får då ett ekvationssystem

$$\begin{cases} 2b = a \\ b^2 + c = 0 \end{cases} \iff \begin{cases} b = \frac{a}{2} \\ c = -b^2 \end{cases} \iff \begin{cases} b = \frac{a}{2} \\ c = -\frac{a^2}{4} \end{cases}$$

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### 2.30

a)

Använd konjugatregeln i täljaren

$$\frac{4x^2 - 4}{2x + 2} = \frac{(2x + 2)(2x - 2)}{2x + 2} = 2x - 2$$

c)

$$\frac{1}{x^2} - \frac{1}{y^2} + \frac{x^2 - y^2}{(xy)^2} = \frac{y^2}{x^2y^2} - \frac{x^2}{x^2y^2} + \frac{x^2 - y^2}{x^2y^2} = \frac{y^2 - x^2 + x^2 - y^2}{x^2y^2} = \frac{0}{x^2y^2} = 0$$

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### 2.31

a)

$$\begin{aligned} \frac{2}{3x + 9} + \frac{x}{x^2 - 9} - \frac{1}{2x - 6} &= \frac{2}{3(x + 3)} + \frac{x}{(x + 3)(x - 3)} - \frac{1}{2(x - 3)} \\ &= \frac{4(x - 3) + 6x - 3(x + 3)}{6(x + 3)(x - 3)} = \frac{4x - 12 + 6x - 3x - 9}{6(x + 3)(x - 3)} \\ &= \frac{7x - 21}{6(x + 3)(x - 3)} = \frac{7(x - 3)}{6(x + 3)(x - 3)} \\ &= \frac{7}{6(x + 3)} \end{aligned}$$

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### 2.32

a)

$$\begin{aligned} \frac{3x - y}{x^2 - 2xy + y^2} - \frac{2}{x - y} - \frac{2y}{(x - y)^2} &= \frac{3x - y}{(x - y)^2} - \frac{2}{x - y} - \frac{2y}{(x - y)^2} \\ &= \frac{3x - y}{(x - y)^2} - \frac{2(x - y)}{(x - y)^2} - \frac{2y}{(x - y)^2} = \frac{3x - y - 2(x - y) - 2y}{(x - y)^2} \\ &= \frac{x - y}{(x - y)^2} = \frac{1}{x - y} \end{aligned}$$

b)

$$\begin{aligned} \frac{a}{a^2 + 4ab + 4b^2} + \frac{2b}{a^2 - 4b^2} &= \frac{a}{(a + 2b)^2} + \frac{2b}{(a + 2b)(a - 2b)} \\ &= \frac{a(a - 2b)}{(a + 2b)^2(a - 2b)} + \frac{2b(a + 2b)}{(a + 2b)^2(a - 2b)} = \frac{a(a - 2b) + 2b(a + 2b)}{(a + 2b)^2(a - 2b)} \\ &= \frac{a^2 + 4b^2}{(a + 2b)^2(a - 2b)} \end{aligned}$$

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**2.33**

$$\begin{aligned} & \frac{1}{(x-1)(x-2)} + \frac{1}{(x-1)(x-3)} + \frac{1}{(x-2)(x-3)} \\ = & \frac{x-3}{(x-1)(x-2)(x-3)} + \frac{x-2}{(x-1)(x-2)(x-3)} + \frac{x-1}{(x-1)(x-2)(x-3)} \\ = & \frac{x-3+x-2+x-1}{(x-1)(x-2)(x-3)} = \frac{3x-6}{(x-1)(x-2)(x-3)} \\ = & \frac{3(x-2)}{(x-1)(x-2)(x-3)} = \frac{3}{(x-1)(x-3)} \end{aligned}$$

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