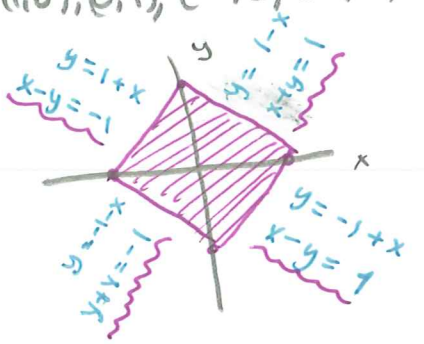
Ex. $D: x^2+2x+y^2 \le 0$

$\Leftrightarrow (x+1)^2 - 1 + y^2 \le 0$

$\Leftrightarrow (x+1)^2 + y^2 \le 1$

$$\begin{cases} x+1 = r\cos\varphi \\ y = r\sin\varphi \end{cases} \Updownarrow \begin{cases} x = r\cos\varphi - 1 \\ y = r\sin\varphi \end{cases}$$

Ex $I: \iint_D \frac{(x+y)^2}{1+(x-y)^2}\,dx\,dy$ D: kvadraten med hörn $(1,0), (0,1), (-1,0), (0,-1)$



Båda integranden och området passar för bytet $\begin{cases} x+y = u \\ x-y = v \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{2}(u+v) \\ y = \frac{1}{2}(u-v) \end{cases}$

$D: \begin{cases} -1 \le x+y \le 1 \\ -1 \le x-y \le 1 \end{cases} \rightarrow E: \begin{cases} -1 \le u \le 1 \\ -1 \le v \le 1 \end{cases}$

$$\frac{d(x,y)}{d(u,v)} = \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2} \Rightarrow \left|\frac{d(x,y)}{d(u,v)}\right| = \frac{1}{2}$$

kostar om ordningen

$$I = \iint_E \frac{u^2}{1+v^2}\,\frac{1}{2}\,du\,dv = \frac{1}{2}\int_{-1}^{1} u^2\,du \int_{-1}^{1} \frac{1}{1+v^2}\,dv =$$

$$\frac{1}{2}\left[\frac{u^3}{3}\right]_{-1}^{1}\left[\arctan v\right]_{-1}^{1} = \frac{1}{2}\left(\frac{1}{3} - \left(-\frac{1}{3}\right)\right)\left(\arctan 1 - \arctan(-1)\right)$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{1}{3}\cdot\frac{\pi}{2} = \boxed{\frac{\pi}{6}}$$