# Determinanter av ordning $n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

det $A$ ska def.
$D_{ij}$, underdeterminanter
def. som tidigare.

$D_{12}$

"Def" (Rekursiv)

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & & & \\ & & & \end{vmatrix} = a_{11}D_{11} - a_{12}D_{12} + a_{13}D_{13} - \cdots + (-1)^{n+1} a_{1n} D_{1n}$$

ex (16, s219)

$$\begin{vmatrix} 2 & 3 & 4 & -1 \\ 1 & -1 & 2 & 0 \\ -2 & 0 & 1 & 1 \\ 2 & 3 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 5 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ -2 & -2 & 5 & 1 \\ 2 & 5 & -5 & -1 \end{vmatrix} = (-1) \begin{vmatrix} 5 & 0 & 1 \\ -2 & 5 & 1 \\ 5 & -5 & 1 \end{vmatrix}$$

+1

-2

5

$$= (-1) \begin{vmatrix} 5 & 0 & -1 \\ -2 & 5 & 1 \\ 5 & -5 & -1 \end{vmatrix} = (-1) \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 0 & 0 & -1 \\ 3 & 5 & 1 \\ 0 & -5 & -1 \end{vmatrix} \end{array} = (-1)(-1)(-15) = -15$$

# Cramers regel

$A\overline{X} = Y$ där $A = [A_1 A_2 A_3]$

$\det A \neq 0$, $\overline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$x_1 = \frac{\det([Y A_2 A_3])}{\det A}$$

$$x_2 = \frac{\det([A_1 Y A_3])}{\det A}$$

$$x_3 = \frac{\det([A_1 A_2 Y])}{\det A}$$

B. $Y = A\overline{X} = [A_1 A_2 A_3]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 A_1 + x_2 A_2 + x_3 A_3$

$\det([Y A_2 A_3]) =$

$\det([x_1 A_1 + x_2 A_2 + x_3 A_3 \;\; A_2 \;\; A_3]) =$

$x_1 \det([A_1 A_2 A_3]) + \cancel{x_2 \det([A_2 A_2 A_3])}^{\,0} + \cancel{x_3 \det([A_3 A_2 A_3])}^{\,0}$

$= x_1 \det A$