

Determinanter av ordning n .

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$\det A$ ska def.

D_{ij} , underdeterminanter
def. som tidigare.

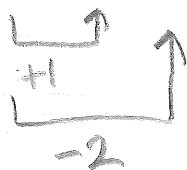
D_{12}

"Def" (Rekursiv)

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = a_{11}D_{11} - a_{12}D_{12} + a_{13}D_{13} - \dots + (-1)^{1+n} a_{1n}D_{1n}$$

ex (16, s219)

$$\begin{vmatrix} 2 & 3 & 4 & -1 \\ 1 & -1 & 2 & 0 \\ -2 & 0 & 1 & 1 \\ 2 & 3 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 5 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ -2 & -2 & 5 & 1 \\ 2 & 5 & -5 & -1 \end{vmatrix} = (-1) \begin{vmatrix} 5 & 0 & 1 \\ -2 & 5 & 1 \\ 5 & -5 & 1 \end{vmatrix}$$



$$= (-1) \begin{vmatrix} 5 & 0 & -1 \\ -2 & 5 & 1 \\ 5 & -5 & -1 \end{vmatrix} = (-1) \begin{vmatrix} 0 & 0 & -1 \\ 3 & 5 & 1 \\ 0 & -5 & -1 \end{vmatrix} = (-1)(-1)(-15) = -15$$

Cramers regel

$$A\mathbf{x} = Y \quad \text{där } A = [A_1 A_2 A_3]$$

$$\det A \neq 0, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1 = \frac{\det([YA_2A_3])}{\det A}$$

$$x_2 = \frac{\det([A_1YA_3])}{\det A}$$

$$x_3 = \frac{\det([A_1A_2Y])}{\det A}$$

$$B. \quad Y = A\mathbf{x} = [A_1 A_2 A_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 A_1 + x_2 A_2 + x_3 A_3$$

$$\det([YA_2A_3]) =$$

$$\det([x_1 A_1 + x_2 A_2 + x_3 A_3 \quad A_2 \quad A_3]) =$$

$$x_1 \det([A_1 A_2 A_3]) + x_2 \det(\cancel{[A_2 A_2 A_3]}) + x_3 \det(\cancel{[A_3 A_2 A_3]})$$

$$= x_1 \det A$$