

# 5-Gradient, Gauss divergence Theorem and Green-Gauss Theorem

5.1) What is a gradient in 1D?

Derivative.

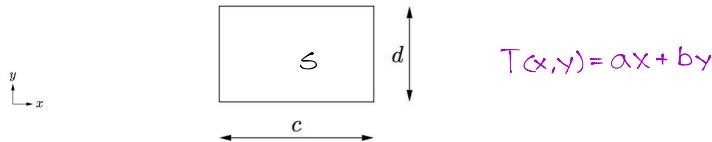
What is Gauss theorem in 1D?

"Analysens huvudsats":  $\int_a^b \frac{df}{dx} dx = f(b) - f(a)$ .

What is Green-Gauss theorem in 1D?

Partial integration:  $\int f \cdot g dx = [f'g] - \int f'g dx$

5.1) To Manu



a) Calculate the integral  $\oint (\nabla T)^T \bar{n} dL$ , where  $\bar{n}$  is normal to the disc.

$$\nabla T = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)^T (ax + by) = (a, b, 0)^T$$

$$\bar{n} = (0, 0, 1)^T$$
$$\Rightarrow (\nabla T)^T \bar{n} = (a, b, 0) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 \Rightarrow \boxed{\oint (\nabla T)^T \bar{n} dL = 0}$$

b) Calculate the divergence of the temperature gradient.

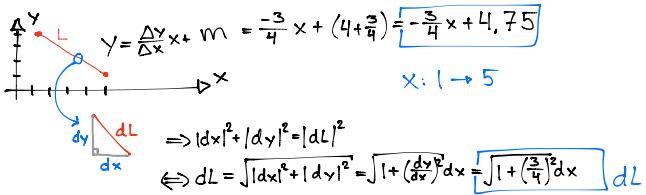
$$\operatorname{div}(\nabla T) = \nabla \cdot (\nabla T) = \nabla \cdot (a, b, 0) = 0$$

c) Could a) been obtained directly from b)?

$$\oint (\nabla T)^T \bar{n} dL = \iint_S \operatorname{div}(\nabla T) dA = 0, \quad \boxed{\text{yes.}}$$

5.3) Calculate the line integral.

$$\begin{aligned}\Phi &= x^2 + y + 10 \\ L &: (1, 4) \rightarrow (5, 1)\end{aligned}$$

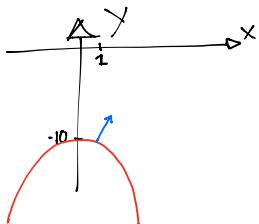


$$\int_L \Phi(x, y) dL = \int_1^5 (x^2 + y + 10) dx = \int_1^5 (x^2 + (-\frac{3}{4}x + 4.75) + 10) \sqrt{1 + (\frac{3}{4})^2} dx = \dots = \boxed{\frac{685}{6}}$$

What happens if we integrate from (5, 1) to (1, 4) instead?

$$-\frac{685}{6} - \int_a^b$$

5.4) A curve in the xy-plane is defined by  $\Phi = x^2 + y + 10 = 0$ . Calculate the normal vector to the curve at (1, -11).



$$\nabla \Phi = (2x, 1)$$

$$\nabla \Phi(1, -11) = (2, 1)$$

We need to normalize:

$$\hat{n} = \frac{(2, 1)}{\sqrt{2^2 + 1^2}} = \boxed{\frac{(2, 1)}{\sqrt{5}}}$$