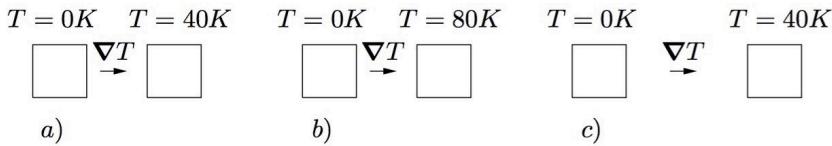


4 - Strong & Weak formulation, 1D-Heat

4.1 Two bodies with different T are located at distance Δx from each other.



Give intuitive answers to the questions below:

o In what direction is heat flowing in a)?

From hot, to cold \Leftrightarrow Left

What happens to ∇T and the flux if the T-diff. is doubled?

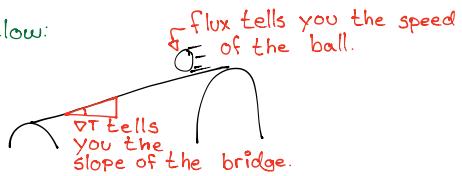
The heat flow will be doubled $\Rightarrow \nabla T$ is doubled and the flux is doubled.
 Height diff. Ball speed.

What happens to ∇T and the flux if the distance is doubled?

Halved, I guess...

o Can you conclude your findings?

Flux $q \propto \Delta T \frac{1}{\Delta x}$, Genius



4.2) Derive the strong & weak form of the 1D heat flow problem.

Lika mycket in som ut!

Stationär värmceledning

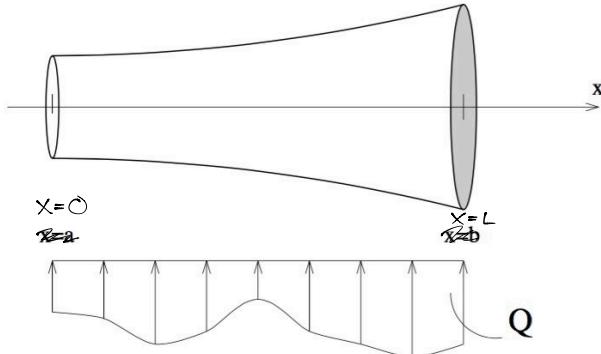
$$\Rightarrow H(x) + \int_x^{x+\Delta x} Q(x) dx = H(x+\Delta x)$$

Mha medelvärdessatsen får vi:

$$\int_x^{x+\Delta x} Q(\eta) d\eta = Q(\xi) \Delta x \quad , \quad \xi \in [x, x+\Delta x]$$

$$\Rightarrow \frac{H(x + \Delta x) - H(x)}{\Delta x} - Q(\epsilon) = 0$$

$$\Delta x \rightarrow 0 \Rightarrow \frac{dH}{dx} - Q = 0$$



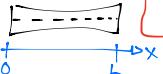
Definition: Värmeflux q .

$$q_t = \frac{H}{A} \Rightarrow \frac{d}{dx}(Aq_t) - Q = 0$$

Gissning: $q = -K \frac{dT}{dx}$ \Rightarrow Konstitutiv lag
 $q = -K \frac{dT}{dx}$ (Fourier)
 Ej huggen i sten.

Randvillkor

Dirchlet / Neumann

$q(x=0) = -(K \frac{dT}{dx})_x=0 = h$	Neumann
Ex. 	Dirchlet
$T(x=L) = g$	

Den starka formen ges av diff.ekvationen + randvillkoren:

svar:

$\frac{d}{dx} (A k \frac{dT}{dx}) + Q = 0, \quad 0 < x < L$
$q(x=0) = -(K \frac{dT}{dx})_{x=0} = h$
$T(x=L) = g$

Nu ska vi ta fram den svaga formen...

Vi börjar med starka formen ovan och utför lite magi,

1. Multiplicera med en viktfunktion

$$V \cdot \left[\frac{d}{dx} (A k \frac{dT}{dx}) + Q \right] = 0$$

Integrate

$$\int_0^L V \left[\frac{d}{dx} (A k \frac{dT}{dx}) + Q \right] dx = 0 \quad (*)$$

$$\text{Put } Y(x) = \phi(x) \cdot \psi(x) \Rightarrow \frac{dy}{dx} = \frac{d\phi}{dx} \psi + \phi \frac{d\psi}{dx} \Rightarrow \int_a^b \left(\frac{d\phi}{dx} \psi + \phi \frac{d\psi}{dx} \right) dx = [\phi(x) \cdot \psi(x)]_a^b$$

$$\text{This can be rewritten as } \int_a^b \phi \frac{d\psi}{dx} dx = [\phi(x) \psi(x)]_a^b - \int_a^b \phi \frac{d\psi}{dx} dx$$

$$\Rightarrow \int_0^L V \frac{d}{dx} (A k \frac{dT}{dx}) dx = \left[V A k \frac{dT}{dx} \right]_0^L - \int_0^L \frac{dv}{dx} A k \frac{dT}{dx} dx$$

We use this in (*):

$$\int_0^L V \left[\frac{d}{dx} (A k \frac{dT}{dx}) + Q \right] dx = \left[V A k \frac{dT}{dx} \right]_0^L - \int_0^L \frac{dv}{dx} A k \frac{dT}{dx} dx + \int_0^L V Q dx = 0$$

$$\Rightarrow \int_0^L \frac{dv}{dx} A k \frac{dT}{dx} dx = \left[V A k \frac{dT}{dx} \right]_0^L + \int_0^L V Q dx = 0$$

$$\text{since } q = -K \frac{dT}{dx} \text{ and BV} \Rightarrow \left[V A k \frac{dT}{dx} \right]_0^L = \left(V A k \frac{dT}{dx} \right)_{x=L} - \left(V A k \frac{dT}{dx} \right)_{x=0} = - (V A q)_{x=L} + (V A)_{x=0} \cdot h$$

Weak form

Finally:

$\int_0^L \frac{dv}{dx} A k \frac{dT}{dx} dx = - (V A q)_{x=L} + (V A)_{x=0} \cdot h + \int_0^L V Q dx$

4.3) The weak form of the uniaxial heat flow problem is given by:

$$\int_0^L \frac{dv}{dx} Ak \frac{dT}{dx} dx = -(vAq)_{x=L} + (vAh)_{x=0} + \int_0^L vQ dx, \quad T(x = L) = g$$

and the weak form is given by

$$\frac{d}{dx} \left(Ak \frac{dT}{dx} \right) + Q = 0, \quad 0 \leq x \leq L$$

$$q(x = 0) = - \left(k \frac{dT}{dx} \right)_{x=0} = h, \quad T(x = L) = g$$

Show that the weak form implies the strong form.

4.4) A rod is subjected to a force as depicted below:

Show that the strong form of the equilibrium eqns. can be written as:

$$\frac{dN}{dx} + b = 0, \quad N = \sigma A$$

