15 - Approximating functions for the FE-method - vector problems

15.1) Displacement field (u_x, u_y)

$$u_x = \sum N_i^e u_{xi} \quad , \quad u_y = \sum N_i^e u_{yi}$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \quad , \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} \quad , \quad \gamma_{xy} = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}$$

Establish the $\mathbb{N}^e$ and $\mathbb{B}^e$ in matrix relations $u = \mathbb{N}^e a^e$, $\varepsilon = \mathbb{B}^e a^e$

$$u = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \quad , \quad a^e = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ \vdots \\ u_{xn} \\ u_{yn} \end{bmatrix} ,$$

$$u_x = N_1^e u_{x1} + N_2^e u_{x2} + \dots N_n^e u_{xn}$$

$$u_y = N_1^e u_{y1} + N_2^e u_{y2} + \dots + N_n^e u_{yn}$$

Lets say $n = 2$.

$$\Rightarrow u = \begin{bmatrix} N_1^e u_{x1} + N_2^e u_{x2} \\ N_1^e u_{y1} + N_2^e u_{y2} \end{bmatrix} = \begin{bmatrix} N_1^e & 0 & N_2^e & 0 \\ 0 & N_1^e & 0 & N_2^e \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{x2} \\ u_{y1} \\ u_{y2} \end{bmatrix}$$

This works for an arbitrary n.

$$\Rightarrow \mathbb{N}^e = \begin{bmatrix} N_1^e & 0 & N_2^e & \dots & N_n^e & 0 \\ 0 & N_1^e & 0 & \dots & 0 & N_n^e \end{bmatrix}$$

$$\mathbb{B}^e = \tilde{\nabla} \mathbb{N}^e \text{ , where } \tilde{\nabla} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$

$$\Rightarrow \mathbb{B}^e = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} N_1^e & 0 & N_2^e & \dots & N_n^e & 0 \\ 0 & N_1^e & 0 & \dots & 0 & N_n^e \end{bmatrix} = \begin{bmatrix} \frac{\partial N_1^e}{\partial x} & 0 & \frac{\partial N_2^e}{\partial x} & 0 & \dots & \frac{\partial N_n^e}{\partial x} & 0 \\ 0 & \frac{\partial N_1^e}{\partial y} & 0 & \frac{\partial N_2^e}{\partial y} & \dots & 0 & \frac{\partial N_n^e}{\partial y} \\ \frac{\partial N_1^e}{\partial y} & \frac{\partial N_1^e}{\partial x} & \frac{\partial N_2^e}{\partial y} & \frac{\partial N_2^e}{\partial x} & \dots & \frac{\partial N_n^e}{\partial y} & \frac{\partial N_n^e}{\partial x} \end{bmatrix}$$