13 - Linear Elasticity

13.1) Strain energy: $W = \frac{1}{2}\sigma^T \cdot \varepsilon$.

In the situation where plane stress applies, show that the out of plane component ε_{zz} does not contribute to the strain energy.

$W = \frac{1}{2}\sigma^T \cdot \varepsilon \quad , \varepsilon = [\varepsilon_{xx} \; \varepsilon_{yy} \; \varepsilon_{zz} \; \gamma_{xy}] , \sigma = [\sigma_{xx} \; \sigma_{yy} \; 0 \; \sigma_{xy}]$

There is no ε_{zz} component...

13.2) The stiffness tensor $\mathbb{D}$ in $\sigma = \mathbb{D}\varepsilon$ is given by:

$$\boldsymbol{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix}$$

a) Derive $\mathbb{D}$ for plane strain conditions.

Plane strain: only ε_{xx}, ε_{yy} & γ_{xy} are non-zero

$$\Rightarrow \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & & \\ & 1-\nu & \\ & & \frac{1}{2}(1-2\nu) \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} + \frac{\alpha E \Delta T}{1-2\nu} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

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