

12 - Stresses and Strains

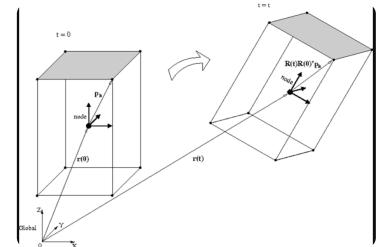
12.1) Derive the strains corresponding to the following displacement fields.

a) $U_x(x,y) = k_1$, $U_y(x,y) = k_2$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x}(k_1) = 0, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} = \frac{\partial}{\partial y}(k_2) = 0, \quad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 0 + 0 = 0$$

$$\Rightarrow \boldsymbol{\varepsilon} = [0 \ 0 \ 0]^T$$

Rigid body motion

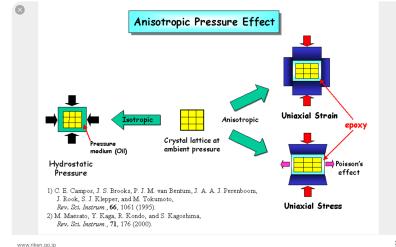


b) $U_x(x,y) = 0$, $U_y(x,y) = k_1 y$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = 0, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} = k_1, \quad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 0 + 0 = 0$$

$$\Rightarrow \boldsymbol{\varepsilon} = [0 \ k_1 \ 0]^T$$

Uniaxial strain

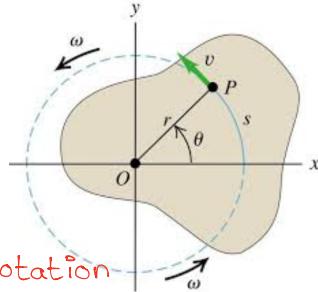


c) $U_x(x,y) = k_1 y$, $U_y(x,y) = -k_1 x$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = 0, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} = 0, \quad \gamma = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = k_1 - k_1 = 0$$

$$\Rightarrow \boldsymbol{\varepsilon} = [0 \ 0 \ 0]^T$$

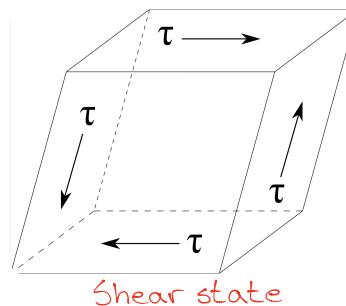
Rigid body rotation



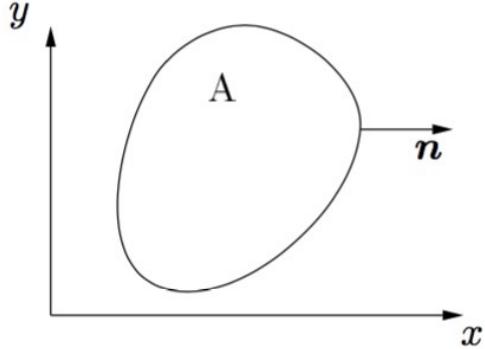
d) $U_x(x,y) = 2k_1 y$, $U_y(x,y) = 0$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = 0, \quad \varepsilon_{yy} = 0, \quad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 2k_1 + 0 = 2k_1$$

$$\Rightarrow \boldsymbol{\varepsilon} = [0 \ 0 \ 2k_1]^T$$



12.2) Consider a disc with uniform thickness t , subjected to a plane stress state.



a) Establish the global equilibrium balance and then derive the local equilibrium.

Read page 240, equilibrium requires that:

$$\int_S t \, ds + \int_V b \, dV = 0$$

Cauchy: $\mathbf{t} = \mathbf{S}^T \mathbf{n}$:

$$\int_S \mathbf{S}^T \mathbf{n} \, ds + \int_V b \, dV = 0$$

$$\text{Gauss th. : } \int_S \mathbf{S}^T \mathbf{n} \, ds = \int_V \text{div}(\mathbf{S}^T) \, dV$$

$$\Rightarrow \int_V (\text{div}(\mathbf{S}^T) + b) \, dV = 0$$

Since this holds for any arbitrary region V , we can write:

$$\boxed{\text{div}(\mathbf{S}^T) + b = 0} \quad \text{Maybe } \mathbf{S} = \mathbf{0}?$$

$$\text{Eq 12.21 : } \boxed{\tilde{\nabla}^T \mathbf{0} + b = 0}$$

$$\text{Eq 12.21 } \boxed{\tilde{\nabla}^T \mathbf{F} + b = 0}$$

b) Determine the traction vector at point P.

$$S = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 12 & 6 & 0 \\ 6 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

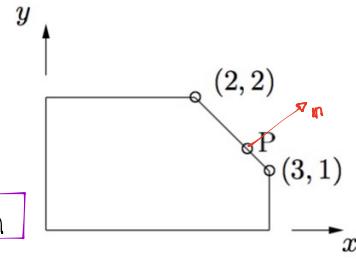
$$\mathbf{n} = \frac{1}{\sqrt{2}}(1, 1, 0)^T$$

$$\mathbf{t} = \frac{1}{\sqrt{2}} \begin{bmatrix} 12 & 6 & 0 \\ 6 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} [18 \ 2 \ 0]^T \quad \mathbf{t}$$

$$\sigma_{nn} = (\mathbf{t} \cdot \mathbf{n})^T \mathbf{n} = \frac{1}{2}(18 + 2) = 10$$

$$\sigma_{nm} = \frac{1}{2}(18 - 2) = 8$$

Traction vector: $\mathbf{t} = 5 \mathbf{n}$



Traction Vectors

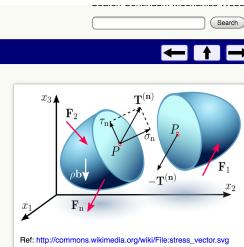
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Introduction

The traction vector, \mathbf{T} , is simply the internal force vector on a cross-section divided by that cross-section's area.

$$\mathbf{T} = \frac{\mathbf{F}_{\text{internal}}}{\text{Area}}$$

So \mathbf{T} has units of stress, like MPa, but it is absolutely a vector, not a stress tensor. So all the usual rules for vectors apply to it. For example, dot products, cross products, and coordinate transforms can be applied.



c) Determine the normal and shear components of the traction vector in P.

$$\text{Normal component: } \sigma_{nn} = \mathbf{n} \cdot \mathbf{t} = \frac{1}{\sqrt{2}}(1 \ 1 \ 0) \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 18 \\ 2 \\ 0 \end{pmatrix} = \frac{1}{2}(18+2) = 10$$

$$\text{Shear component: } \sigma_{nm} = \mathbf{n} \cdot \mathbf{t} = \frac{1}{\sqrt{2}}(1 \ 1 \ 0) \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 18 \\ 2 \\ 0 \end{pmatrix} = \frac{1}{2}(18-2) = 8$$

↑ Randvektor

