

Tenta 28/8 - 12

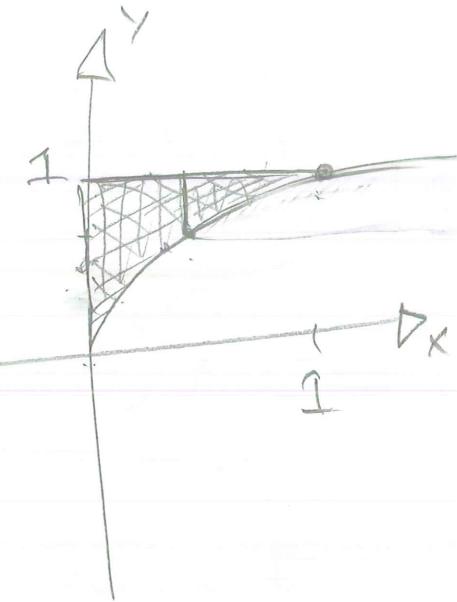
□



$$\iint_D e^{y^3} dx dy =$$

$$\int_0^1 \left(\int_0^{y^2} e^{y^3} dx \right) dy = \int_0^1 e^{y^3} [x]_0^{y^2} dy$$

$$= \int_0^1 y^2 \cdot e^{y^3} dy = \left[\frac{1}{3} e^{y^3} \right]_0^1 = \boxed{\frac{1}{3}(e^1 - 1)}$$



2

$$f'_x + 2f'_y = 5$$

$$f(x,y) = x^2 + x \quad \text{p. 20} \quad x - y = 0$$

$$\Rightarrow \boxed{f(x,y) = x^2 + x}$$

ezpz - No need to do this.

3

$$a) P: \left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$$

$$l = x^2 + y^2 + z^2 = f$$

$$\nabla f = (2x, 2y, 2z) \Rightarrow \nabla f\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right) = \boxed{(1, \sqrt{2}, 1)}$$

$$(1, \sqrt{2}, 1)\left(\frac{1}{2} - x, \frac{1}{\sqrt{2}} - y, \frac{1}{2} - z\right) = 0$$

$$= \frac{1}{2} - x + 1 - \sqrt{2}y + \frac{1}{2} - z = 0$$

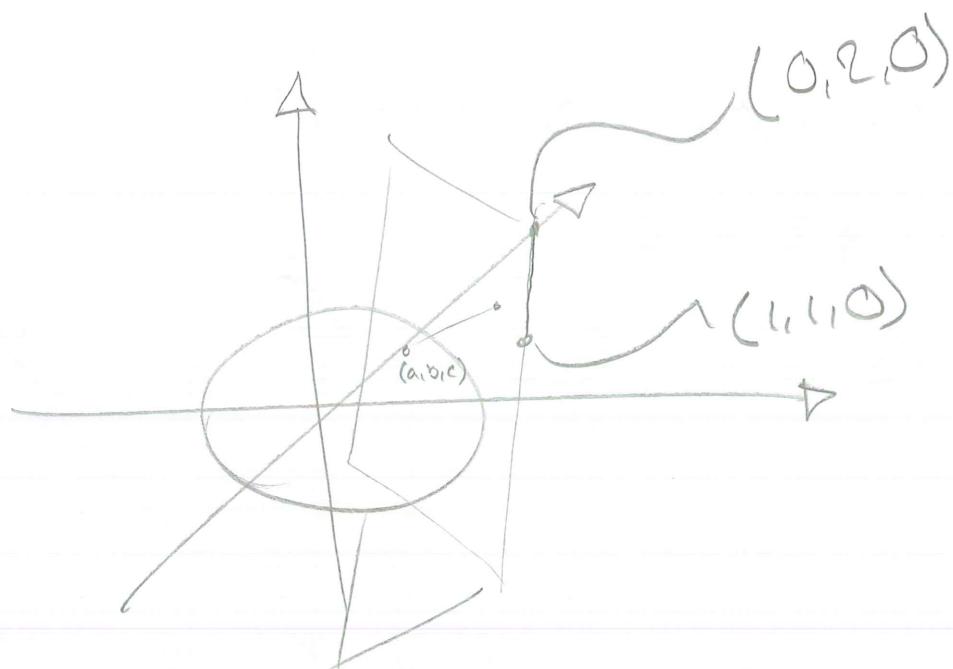
$$\boxed{-x - \sqrt{2}y - z + 2 = 0}$$

$$\bullet (a, b, c) = \\ = \left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$$

$$\bullet (x, y, z)$$

3

b)



$$f: x^2 + y^2 + z^2 = 1$$

$$\nabla f = (2x, 2y, 2z) = (x, y, z) \Rightarrow \nabla f(a, b, c) = (a, b, c)$$

$$(x-a, y-b, z-c)(a, b, c) = 0$$

$$\Rightarrow ax - a^2 + by - b^2 + cz - c^2 = 0$$

$$\Leftrightarrow ax + by + cz = a^2 + b^2 + c^2 = 1$$

$$P_1(0, 2, 0) \Rightarrow 2b = 1 \Leftrightarrow b = \frac{1}{2}$$

$$P_2(1, 1, 0) \Rightarrow a + b = 1 \Rightarrow a = \frac{1}{2}$$

$$c = \pm \sqrt{1 - a^2 - b^2} = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

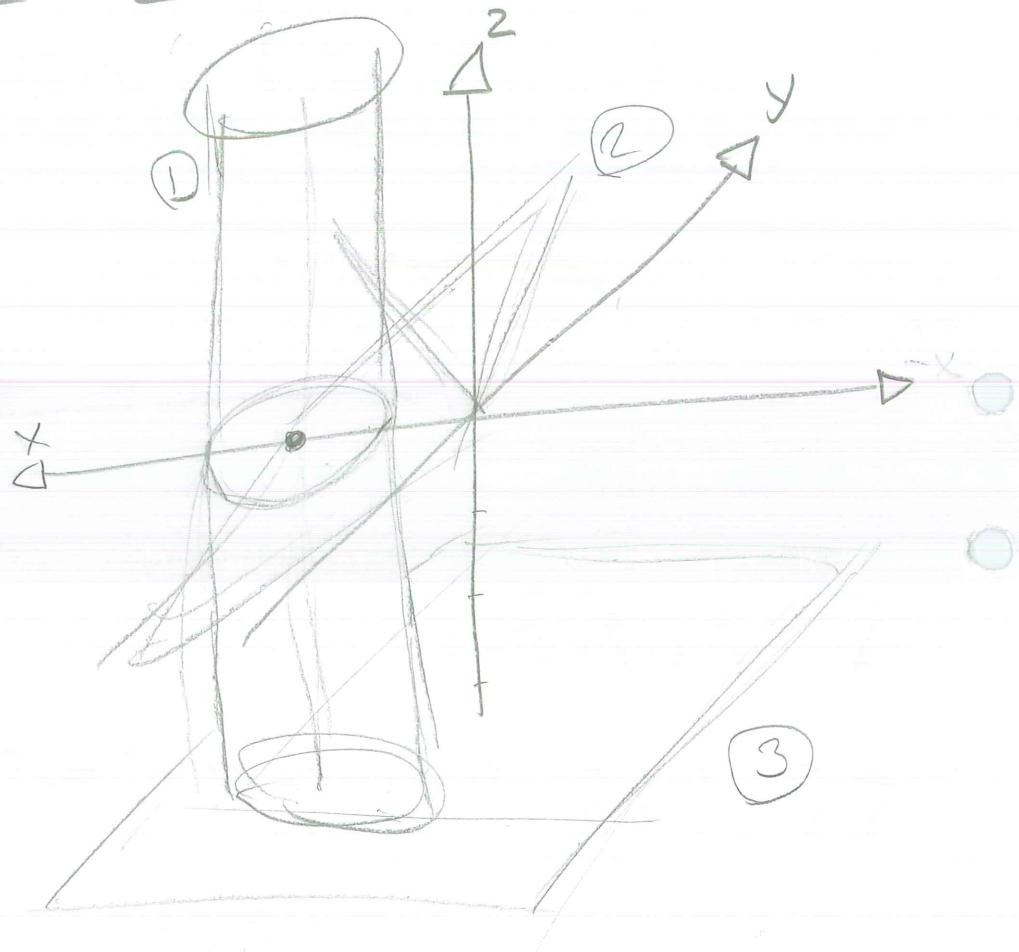
$$\boxed{1: \frac{1}{2}x + \frac{1}{2}y \pm \frac{1}{\sqrt{2}}z = 1}$$

4

$$\textcircled{1} \quad x^2 - 4x + y^2 + 3 = 0 \Rightarrow (x-2)^2 + y^2 = 1$$

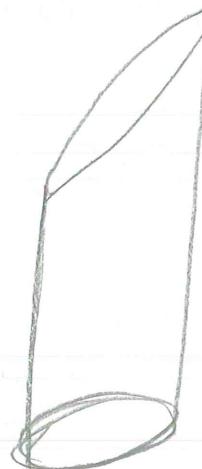
$$\textcircled{2} \quad -x + y + z = 2$$

$$\textcircled{3} \quad z = -3$$



$$\iiint_E 1 \, dx \, dy \, dz =$$

$$\iint_D \left(\int_{-3}^{2+x-y} 1 \, dz \right) dx \, dy =$$



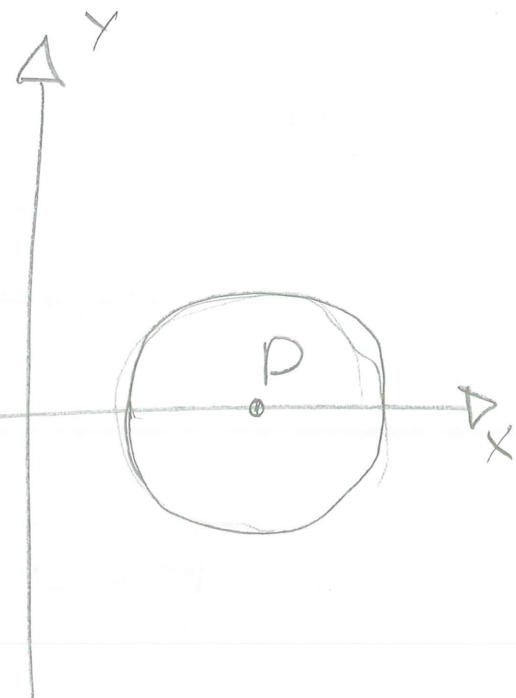
$$-\iint_D (x+y+z) \, dx \, dy = \iint_D (x-y+5) \, dx \, dy$$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$dx dy = r dr d\theta$$

$$r: \mathbb{O} \rightarrow \mathbb{I}$$

$$\theta: \mathbb{O} \rightarrow 2\pi$$



$$\int_0^r \int_0^{2\pi} (e + r \cos \theta + r \sin \theta + 5) r dr d\theta$$

$$= \int_0^{2\pi} \left[r^2 + \frac{r^3}{3} \cos \theta + \frac{r^3}{3} \sin \theta + \frac{5r^2}{2} \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \left(\frac{r^2}{2} + \frac{1}{3} \cos \theta + \frac{1}{3} \sin \theta + \frac{5}{2} \right) d\theta = \left[\frac{r^2}{2} + \frac{1}{3} \sin \theta + \frac{1}{3} \cos \theta \right]_0^{2\pi}$$

$$= 7\pi + 0 - \frac{1}{3} + \frac{1}{3} = \boxed{7\pi}$$

5

a) Greens formel

$$\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

" 1 "

om $P=0, Q=x:$

$$\int_C P dx + Q dy = \iint_D (1 - 0) dx dy =$$

$$= \iint_D 1 dx dy = \text{Area on } D \quad \#$$

b)

$$\begin{cases} x = 2 \cos t + \cos 2t \\ y = 2 \sin t - \sin 2t \end{cases} \quad t: 0 \rightarrow 2\pi$$

$$\iint_D 1 dx dy = \int_C x dy = \int_0^{2\pi} (2 \cos t + \cos 2t)(2 \sin t - \sin 2t)$$

$$-\frac{1}{2} \left[\int_C -y dx + x dy \right]_0^{2\pi} \quad \rightarrow$$

$$\frac{1}{2} \int_0^{\pi} (-2\sin t + \sin 2t)(-2\sin t - \sin 2t)$$

$$+ (2\cos t + \cos 2t)(2\cos t - \cos 2t) dt$$

$$= \frac{1}{2} \int_0^{\pi} 4\sin^2 t + 4\cancel{\sin t \sin 2t} + \cancel{8\sin t \sin 2t} - 2\sin^2 2t$$

$$+ 4\cos^2 t - 4\cos t \cos 2t + 2\cos t \cos 2t - \cancel{2\cos^2 2t} =$$

$$= \frac{1}{2} \int_0^{\pi} 2\sin^2 t + 2\cos^2 t + \cancel{2\sin t \sin 2t} - \cancel{2\cos t \cos 2t} =$$

$$= \int_0^{2\pi} (1 - \cos t \cdot \cos 2t) dt + \boxed{\sin t \sin 2t} = \boxed{2.51}$$

$$= \int_0^{2\pi} 1 - \cancel{\cos t} - \cos t \sin t + 2\sin^2 t \cos t dt =$$

$$= \int_0^{2\pi} 1 - \cos^3 t dt + \left[\frac{1}{3} \sin^3 t \right]_0^{2\pi} = \boxed{2.51}$$

6

$$h(x, y, z) = x \ln x + y \ln y + z \ln z$$

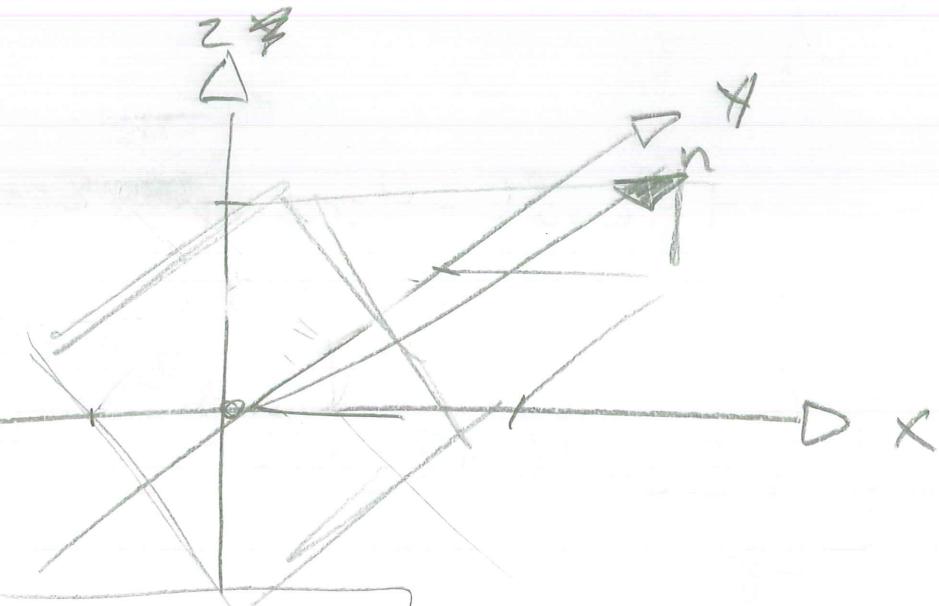
$f_1: x + y + z = 1$

$$\nabla h = (\ln x + 1, \ln y + 1, \ln z + 1)$$

$$3e^{x-1} = 1$$

$$x-1 = \ln \frac{1}{3}$$

$$x = \ln \frac{1}{3} + 1$$



$$\nabla h = 0 \Rightarrow \left\{ x = \frac{1}{e} = y = z \right\} \text{ ej i planet.}$$

$$\nabla f = (1, 1, 1)$$

$$\lambda(1, 1, 1) = (\ln x + 1, \ln y + 1, \ln z + 1) \quad \text{satt in!}$$

$$\begin{cases} \ln x = \lambda - 1 \\ \ln y = \lambda - 1 \\ \ln z = \lambda - 1 \end{cases} \Rightarrow \begin{cases} x = e^{\lambda-1} \\ y = e^{\lambda-1} \\ z = e^{\lambda-1} \end{cases} = \begin{cases} \frac{1}{e^{\lambda-1}} \\ \frac{1}{e^{\lambda-1}} \\ \frac{1}{e^{\lambda-1}} \end{cases} = \begin{cases} \frac{1}{e^{\lambda-1}} \\ \frac{1}{e^{\lambda-1}} \\ \frac{1}{e^{\lambda-1}} \end{cases}$$

$$h = -\ln 3$$

$$h(1, 0, 0) = \ln 1 = 0$$

$$h(0, 1, 0) = 0$$

$$h(0, 0, 1) = 0$$

ln

$$h(1-t, t, 0) = (1-t) \ln(1-t) + t \cdot \ln t$$

$$h_t' = -\ln(1-t) - (1-t) \frac{1}{1-t} + \ln t + t \cdot \frac{1}{t} =$$

$$= -\ln(1-t) + \ln t = \ln \frac{t}{1-t} = 0$$

$$\Rightarrow \frac{t}{1-t} = 1 \Leftrightarrow t = 1-t \Leftrightarrow t = \frac{1}{2}$$

$$h\left(\frac{1}{2}, \frac{1}{2}, 0\right) = \left(1-\frac{1}{2}\right) \ln\left(1-\frac{1}{2}\right) + \frac{1}{2} \ln \frac{1}{2} =$$

$$= \frac{1}{2} \ln \frac{1}{2} + \frac{1}{2} \ln \frac{1}{2} = \ln \frac{1}{2} = \boxed{-\ln 2}$$

~~Ergebnis~~

Eftersom \circ ej är

positiv behöver man

inte räkna med randen!

$$\text{Minsta: } -\ln 3$$