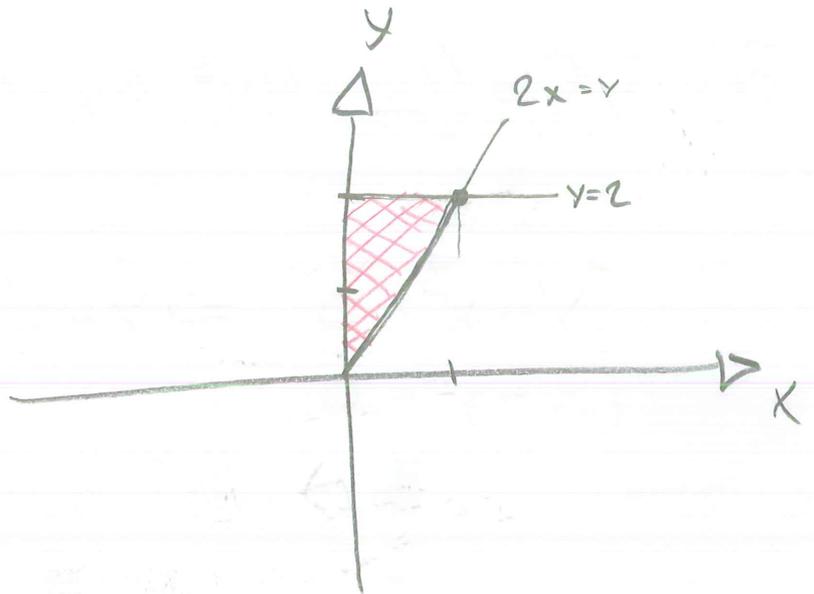


Tenta 21/5-12

1 $\iint_D e^{x^2} dx dy$



$e^{y/2}$

$\int_0^2 \left(\int_0^{y/2} e^{x^2} dx \right) dy =$

$= \int_0^2 \left[x e^{x^2} \right]_0^{y/2} dy = \int_0^2 \frac{y}{2} e^{y^2} dy = \left[\frac{1}{4} e^{y^2} \right]_0^2 = \frac{1}{4}(e^4 - 1)$

2 $\int_x^x f(x)$

NEJ, ez pz.

3

$$f(x, y) = 4xy - x^4 - y^4$$

$$a) \nabla f = (4y - 4x^3, 4x - 4y^3) = (0, 0)$$

$$\begin{cases} y - x^3 = 0 \\ x - y^3 = 0 \end{cases} \Rightarrow y = x^3$$

$$\Delta \Rightarrow x - (x^3)^3 = 0 \Leftrightarrow x(1 - x^8) = 0$$

$$x_1 = 0, \quad x_2 = 1, \quad x_3 = -1$$

$$y_1 = 0, \quad y_2 = 1, \quad y_3 = -1$$

$$f(0, 0) = 0$$

$$f(1, 1) = 4 - 1 - 1 = 2$$

$$f(-1, -1) = 4 - 1 - 1 = 2$$

$$f''_{xx} = -12x^3$$

$$f''_{xy} = 4$$

$$f''_{yy} = -12y^3$$

$$Q(h, k) = f''_{xx}(a, b) \cdot k^2 + 2f''_{xy}(a, b) \cdot kh + f''_{yy}(a, b) h^2$$

P (0, 0): $Q(h, k) = 0 + 8kh + 0 = \boxed{8kh}$ indefinit!
sadel

P (1, 1): $Q(h, k) = -12k^2 + 8kh - 12h^2 =$

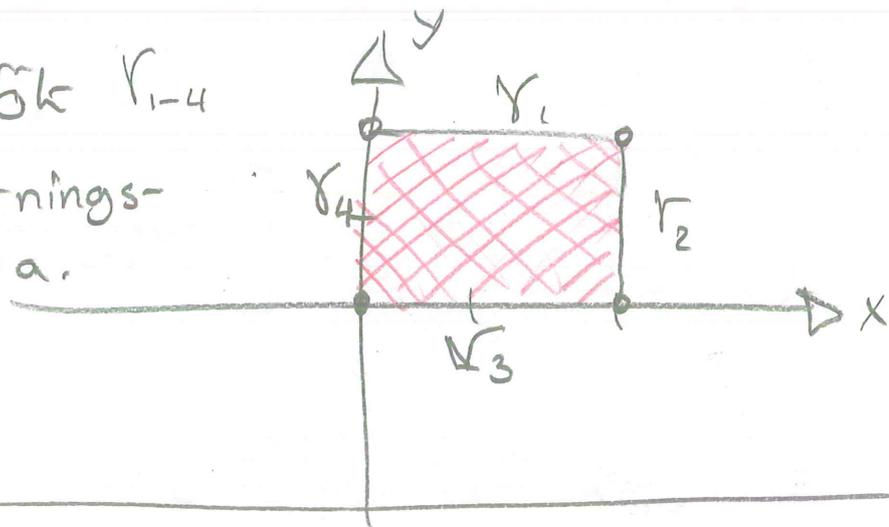
$$= -4(3k^2 + 2kh - 3h^2) =$$

$$= -4((k+h)^2 + 2k^2 + 2h^2) < \boxed{0}$$
negdef!
maxvärde.

P (-1, -1): $Q(h, k) = 12k^2 + 8kh + 12h^2 = 4((h+k)^2 + k^2 + h^2) > 0$
negdef!

Extrempunkter: (1, 1) och (-1, -1)!

3) b) Undersök r_{1-4}
 och skärnings-
 punkterna.



4

$$\int_{\gamma} P dx + Q dy = U(b_1, b_2) - U(a_1, a_2)$$

$$\gamma: (a_1, a_2) \rightarrow (b_1, b_2)$$

$$(P, Q) = \left(\frac{dU}{dx}, \frac{dU}{dy} \right)$$

$$U(x, y) = U(x(t), y(t))$$

$$\frac{d}{dt} U(x(t), y(t)) = \frac{dU}{dx} \cdot \frac{dx}{dt} + \frac{dU}{dy} \cdot \frac{dy}{dt} =$$

$$= P \frac{dx}{dt} + Q \frac{dy}{dt}$$

$$\text{Satz: } \int_{\gamma} P dx + Q dy = \int_s^t \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt =$$

$$\int_s^t \frac{d}{dt} U(x(t), y(t)) dt = \left[U(x(t), y(t)) \right]_s^t = U(x(t), y(t)) - U(x(s), y(s))$$
$$= U(a_2, b_2) - U(a_1, b_1) \quad \# \text{ezpz.}$$

4

$$b) (P, Q) = (2xy^3, 1 + 3x^2y^2)$$

• Inga diskontinuitetspunkter!

$$\frac{dP}{dy} = 6xy^2, \quad \frac{dQ}{dx} = 6xy^2$$

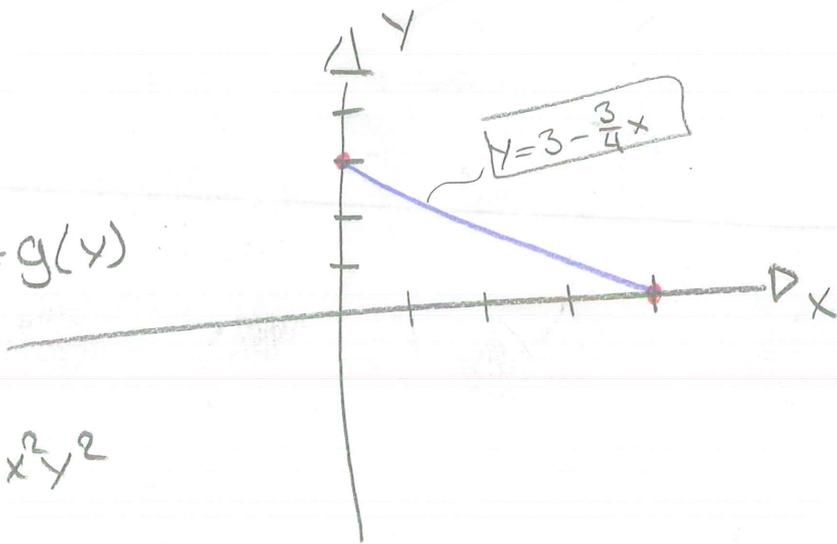
JA!

$$I = \int_{\gamma} 2xy^3 dx + (1 + 3x^2y^2) dy$$

$$\gamma: \cos^2(x, y) = 1 \quad (0, 3) \rightarrow (4, 0)$$

(Uåtgoberoende)

$$\frac{dU}{dx} = 2xy^3 \Rightarrow U = x^2y^3 + g(y)$$



$$\frac{dU}{dy} = 3x^2y^2 + g'(y) = 1 + 3x^2y^2$$

$$\Rightarrow g(y) = y \Rightarrow U(4, 0) - U(0, 3) = 0 + 0 - (0 + 3) = -3$$

5

$$z = 180 - x^2 - 2y^2$$

$$\gamma: (8, -5, 0) \rightarrow (5, 4, 0)$$

$$\bar{v}_\gamma = (-3, 9) \frac{1}{10}$$

$$f'_\gamma = \nabla f \cdot \bar{v}_\gamma$$

$$\nabla f = (-2x, -4y)$$

$$-3x + 9y = 0$$

$$-24 - 45 = 0$$

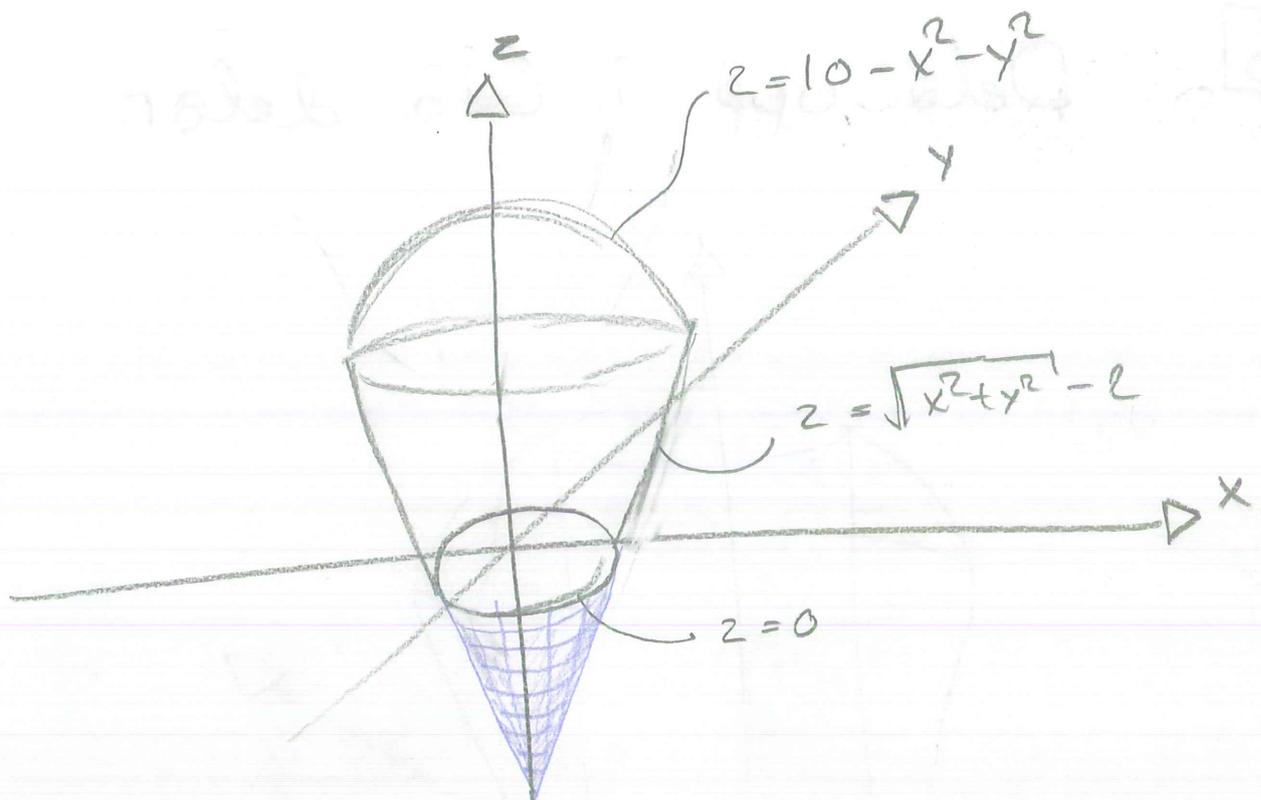
$$-69 = 0$$

$$-x + 3y = 33$$

$$f'_\gamma = -2(x, 2y)(-3, 9) \frac{1}{10} = -\frac{1}{5}(-3x + 18y) =$$

$$= -\frac{3}{5}(-x + 6y)$$

6

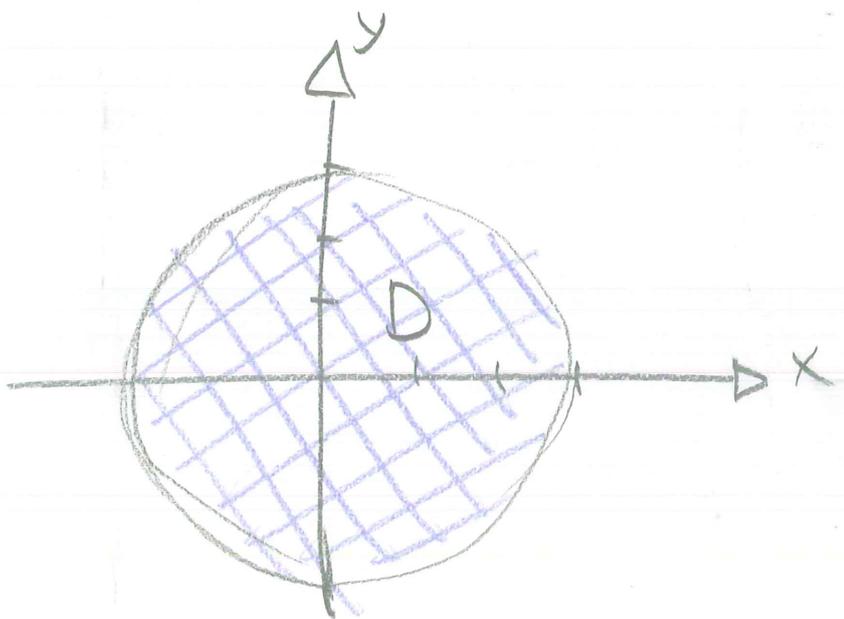


$$10 - x^2 - y^2 = \sqrt{x^2 + y^2} - 2$$

$$10 - r^2 = r - 2 \quad \text{where } r = \sqrt{x^2 + y^2}$$

$$r^2 + r - 12 = 0 \Rightarrow r = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{48}{4}} = -\frac{1}{2} \pm \frac{7}{2}$$

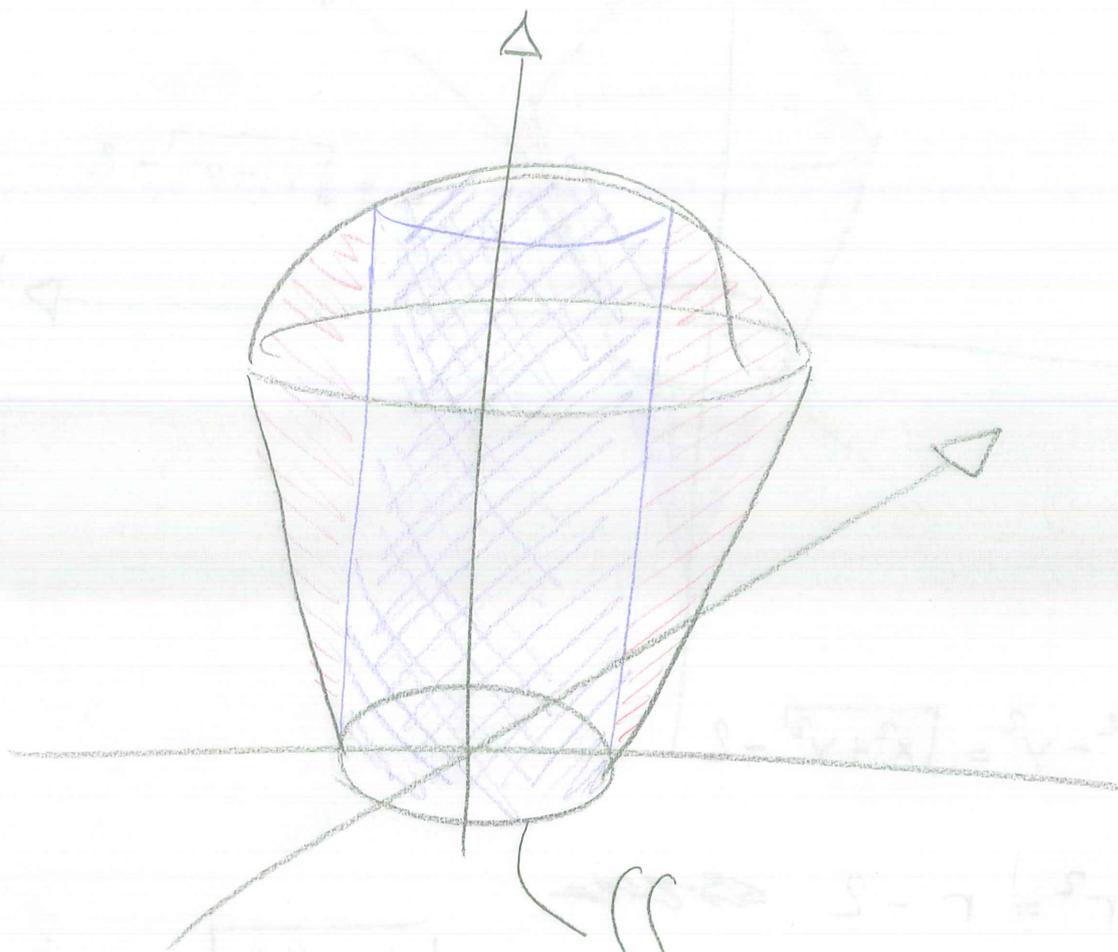
$r_1 = 3$ \Rightarrow



~~ff~~

6

Delar upp i två delar



\iint_D

$\vec{c} = (7)$