Tenta 18/12 - 12

1

$P(x,y) = (x+y)^2 + y^2 = u^2 + v^2$ **a**

$R(x,y) = (2x+3y)^2 = u^2$ **c**

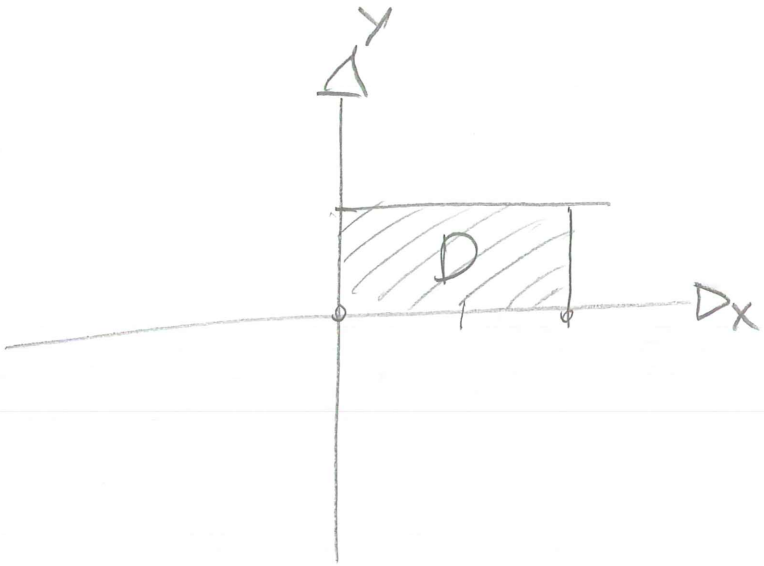
$Q(x,y) = (x+2y)^2 - y^2 = u^2 - v^2$ **b**

$S(x,y) = -((x+y)^2 + 3y^2) = -u^2 - v^2$ **d**

2

$D = \{(x,y);\ 0 \leq x \leq 2,\ 0 \leq y \leq 1\}$

$f(x,y) = 1 + (x-y)^2$



#ezpz

3

$f: \mathbb{R}^2 \to \mathbb{R}$

$g: \mathbb{R}^3 \to \mathbb{R}$

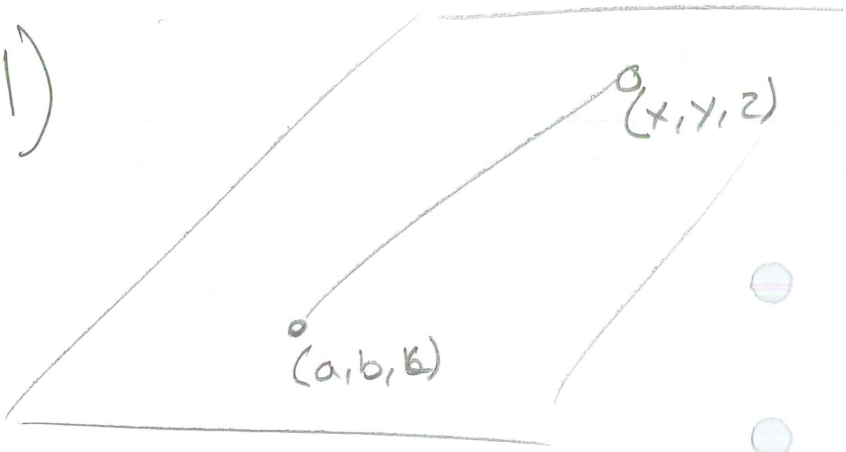
$f(a,b) = c$, $g(a,b,c) = 0$

$g(a,b,f(a,b)) = 0$

$z = f(x,y) \Leftrightarrow 0 = f(x,y) - z$

$h(x,y,z) = f(x,y,z) - z$

$\nabla h = (f'_x, f'_y, -1)$



Så $(f'_x, f'_y, -1)(x-a, y-b, z-c) = 0$

b) $P: (1,1,1)$

$x^2+y^2-z^2-1=0 \quad f = x^2+y^2-z^2$

$\nabla f = (2x, 2y, -2z)$

$\nabla f(1,1,1) = (2, 2, -2)$

$\pi: (2,2,-2)(x-1, y-1, z-1) = 0$

$\Rightarrow 2x-2+2y-2-2z+2 = 0$

$\boxed{\pi: x+y-z-1=0}$

$\nabla f = k \cdot \nabla \pi$

$(2x, 2y, 2z) = k(1,1,-1)$

$$\begin{cases} x = \frac{k}{2} \\ y = \frac{k}{2} \\ z = \frac{k}{2} \end{cases} \rightarrow \text{dă} \quad x = y = z$$

$x+x-x-1=0 \Rightarrow \boxed{x=y=z=1}$

4 $f''_{xx} - f''_{yy} = 8$

$$\frac{df}{dx}\cdot\frac{df}{dx} = \frac{d^2f}{dx^2}$$

$$\begin{cases} s = x+y \\ t = x-y \end{cases}$$

$$f'_x = f'_s \cdot 1 + f'_t \cdot 1$$

$$f'_y = f'_s - f'_t$$

$$f''_{xx} = f''_{ss} + f''_{st} + f''_{ts} + f''_{tt}$$

$$f''_{yx} = f''_{ss} - f''_{st} + f''_{st} + f''_{tt} =$$

$f = x^2$

$\frac{df}{dx} = 2x$

$\frac{d^2f}{dx^2} = 2 = 4x^2$

$$f''_{ss} + 2f''_{st} + f''_{tt} - f''_{ss} + f''_{tt} = 8$$

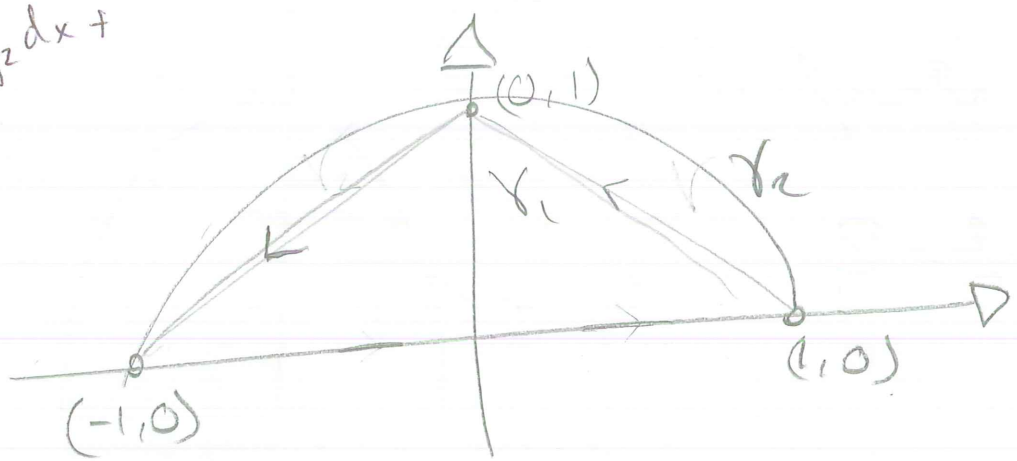
$$f''_{st} = 4 \Rightarrow f'_s = 4t + g(s)$$

$$f(s,t) = 4st + G(s) + h(t)$$

5

a) $F(x,y) = \left(\underbrace{\frac{y}{x^2+y^2}}_{P}, \underbrace{\frac{-x}{x^2+y^2}}_{Q} \right)$

$\int_{\gamma_1} \frac{y}{x^2+y^2} dx +$



$$\frac{dP}{dy} = \frac{dQ}{dx}$$

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \Rightarrow \begin{cases} dx = -\sin t\, dt \\ dy = \cos t\, dt \end{cases} \quad t: 0 \to \pi$$

$$\int_{\gamma_1} = \int_{\gamma_2} = \int_0^{\pi} -1\, dt = \boxed{-\pi}$$

b)

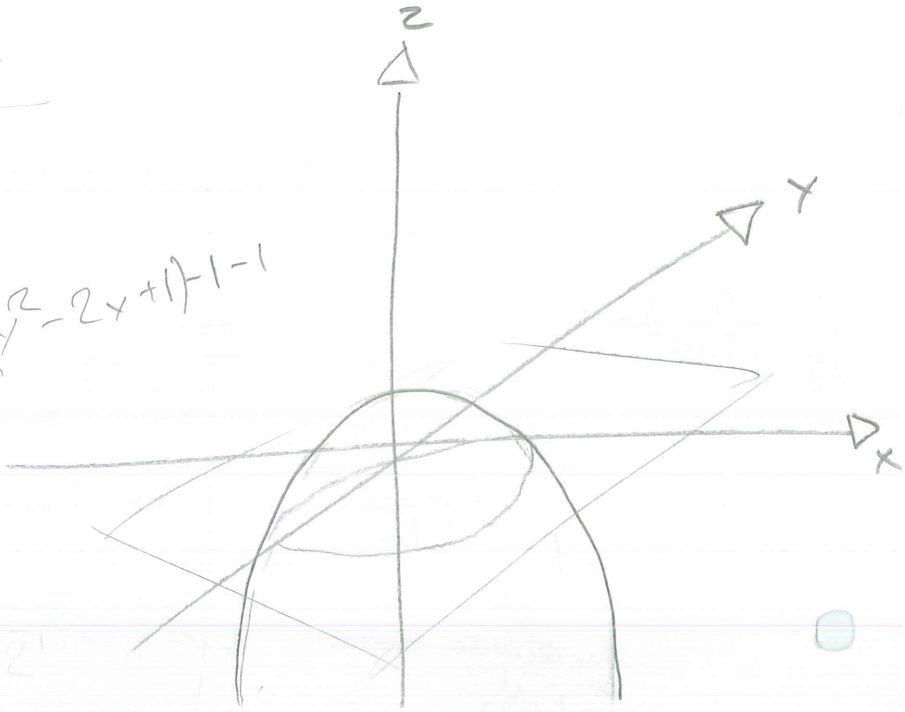
$$z = 1 - x^2 - y^2$$

$$\pi : 2y + z = 0$$

$$-2y = 1 - x^2 - y^2$$

$$(y^2 - 2y + 1) + 1 - 1$$

$$(y-1)^2 + x^2 = 2$$



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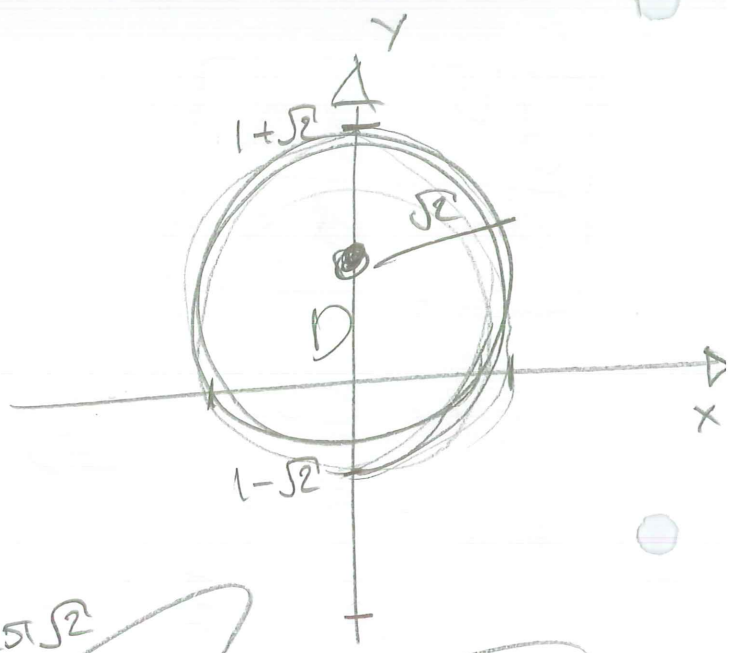
$$\iiint_K 1 \, dx\,dy\,dz =$$



$$= \iint_D \left(\int_{-2y}^{1-x^2-y^2} 1 \, dz \right) dx\,dy =$$

$$\iint_D 1 - x^2 - y^2 - 2y \, dx\,dy = \int_0^{2\pi} \int_0^{\sqrt{2}} (1 - r^2 - 2r\sin\theta) \cdot r \, dr\,d\theta$$

$$= \int_0^{2\pi} \left[\frac{r^2}{2} - \frac{r^4}{4} - \frac{2r^3}{3} \sin\theta \right]_0^{\sqrt{2}} = \int_0^{2\pi} 1 - 1 - \frac{2\sqrt{2}^3}{3} \sin\theta \, d\theta$$

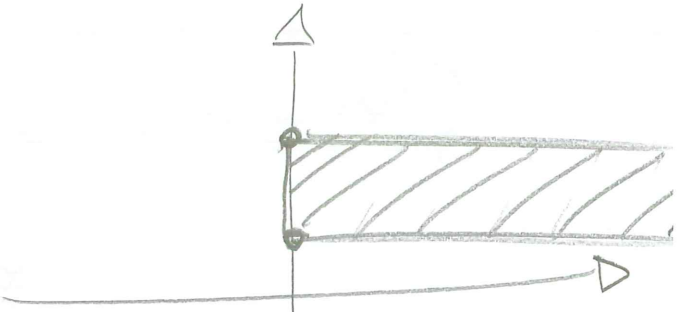
$$\frac{4\sqrt{2}}{3} \left[+ \cos\theta \right]_0^{2\pi} = \boxed{2\pi}$$

$$\begin{cases} x = r\cos\theta \\ y = 1 + r\sin\theta \end{cases}$$

$$\iint_D (2 - r^2)\, r = 2\pi \left[r^2 - \frac{r^4}{4} \right]_0^{\sqrt{2}} = 2\pi \left(2 - \frac{4}{4}\right) = \boxed{2\pi}$$

6

a) $\iint_D e^{-xy}\, dx\, dy$



$$\int_0^{\infty} \left(\int_1^2 e^{-xy}\, dy \right) dx = \int_0^{\infty} \left[-\frac{1}{x} e^{-xy} \right]_1^2 dx =$$

$$\int_0^{\infty} -\frac{1}{x} e^{-2x} + \frac{1}{x} e^{-x}\, dx = \int_0^{\infty} \frac{1}{x} \left(e^{-x} - e^{-2x} \right) dx =$$

$$\int_1^2 \left(\int_0^{\infty} e^{-xy}\, dx \right) dy = \int_1^2 \left[-\frac{1}{y} e^{-xy} \right]_0^{\infty} dy = \int_1^2 + \frac{1}{y} = -\ln 2 + \ln 1 =$$

$$= \boxed{-\ln 2}$$

ezpz

6

b) $$\int_0^\infty \frac{e^{-x} - e^{-2x}}{x} = \boxed{\ln 2}$$

c) $$\int_0^\infty \frac{f(x) - f(2x)}{x} dx$$

$f(x) \to 0$ dă $x \to \infty$

$$\left[\begin{matrix} t = 2x \\ dx = \frac{1}{2} dt \end{matrix}\right] = \int_0^\infty \frac{f\left(\frac{t}{2}\right) - f(t)}{t} \cdot 2 \cdot \frac{1}{2} dt$$