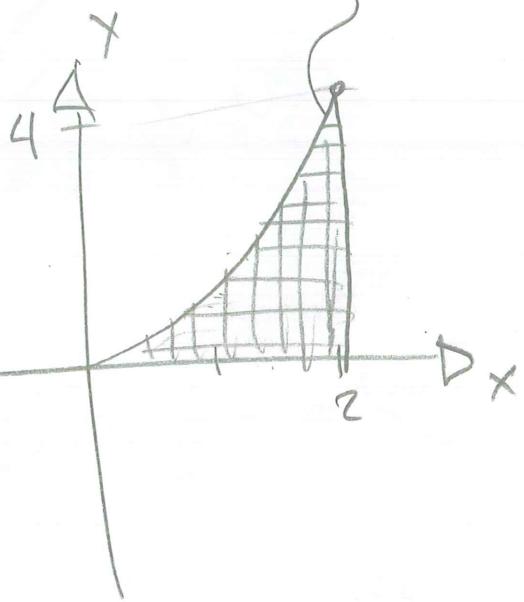


# Tenta 10/4-12

11

$$\iint_D \frac{x}{1+y} dx dy$$

$$y = x^2$$



$$\int_0^2 \left( \int_0^{x^2} x \cdot \frac{1}{1+y} dy \right) dx =$$

$$= \int_0^2 x \left[ \ln(1+y) \right]_0^{x^2} = \int_0^2 x \cdot \ln(1+x^2) -$$

$$\int_0^4 \left( \int_{\sqrt{y}}^2 \frac{1}{1+y} \times dx \right) dy = \int_0^4 \frac{1}{1+y} \cdot \left[ \frac{x^2}{2} \right]_{\sqrt{y}}^2 dy =$$

$$= \int_0^4 \frac{2 - \frac{y}{2}}{1+y} = \left[ 2 \ln(1+y) - \frac{y}{2} + \frac{1}{2} \ln(1+y) \right]_0^4 = \boxed{\frac{5}{2} \ln 5 - 2}$$

2

a) Inga diskontinuitetspunkter.

5)

$$I = \int -y \frac{dx}{x^2+y^2} + x \frac{dy}{x^2+y^2},$$

~~$$\frac{d\phi}{dy} = \frac{-y}{x^2+y^2} \Rightarrow \text{differential}$$~~

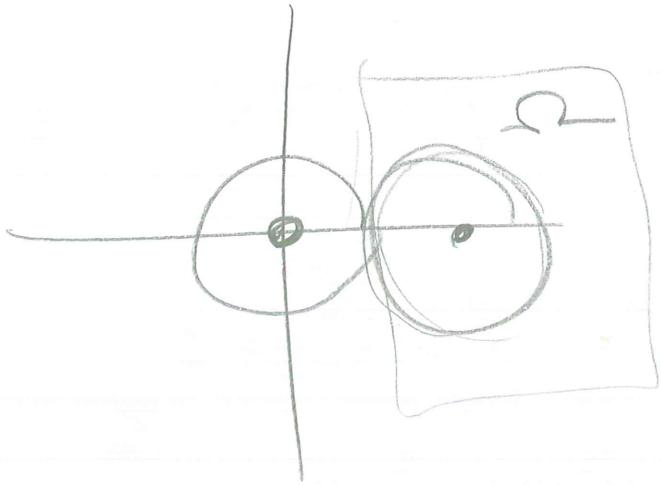
~~$$\phi = -\frac{1}{2} \ln(x^2+y^2) + g(x)$$~~

~~$$\frac{dC}{dx} = -\frac{x}{x^2+y^2} + g'(x) \neq -\frac{x}{x^2+y^2}$$~~

2

b)

$$\begin{cases} x = \cos \theta \\ y = \sin \theta \end{cases}$$



$$\begin{cases} dx = -\sin \theta d\theta \\ dy = \cos \theta d\theta \end{cases}$$

$$I = \int_0^{2\pi} \left( \frac{\sin^2 \theta}{1} + \frac{\cos^2 \theta}{1} \right) d\theta = \int_0^{2\pi} 1 d\theta = [2\pi]$$

c)  $\gamma$  innesluter ej diskontinuitetspunkten.  $\int_\gamma$  är alltså noll.

(samma start-/slutpunkt).

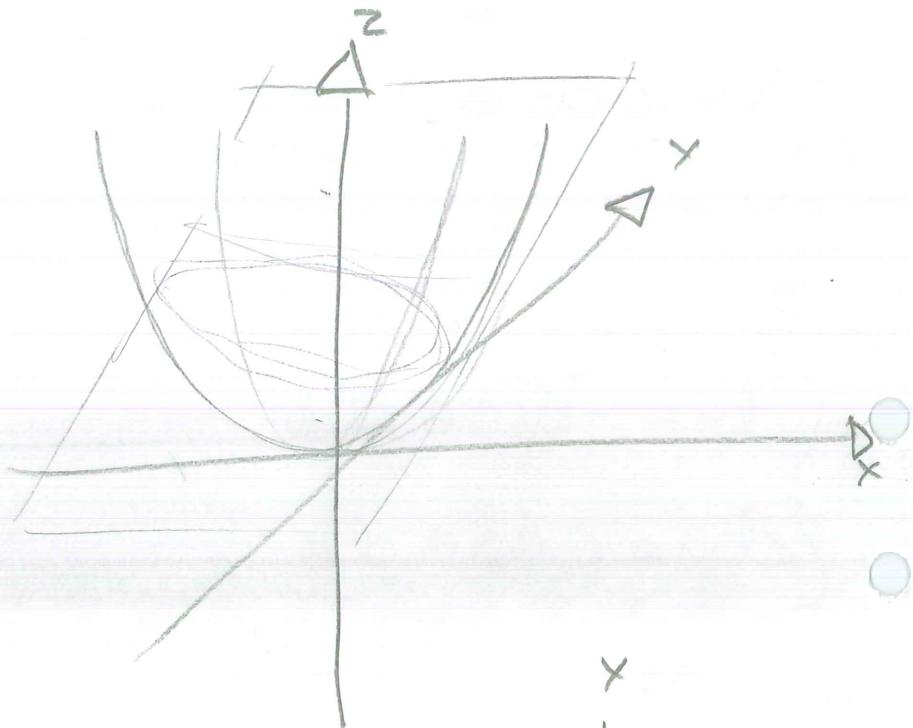
3

$$z = x^2 + y^2 \quad z = 1 - 2y \quad (n = (0, 2, 1))$$

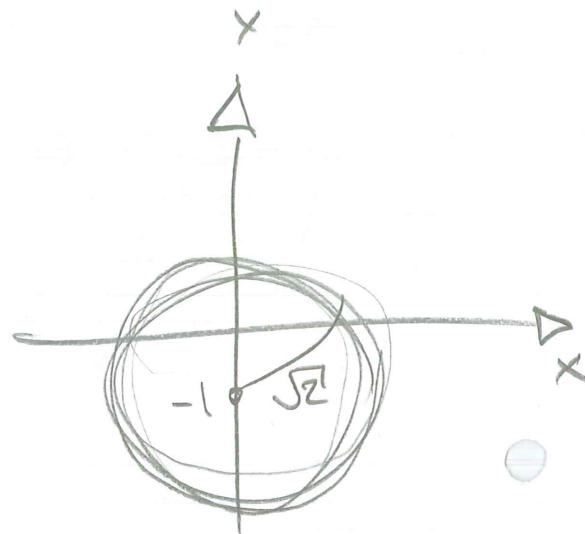
$$x^2 + y^2 - 1 + 2y = 0$$

$$x^2 + (y+1)^2 = 2$$

$$\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y+1}{\sqrt{2}}\right)^2 = 1$$

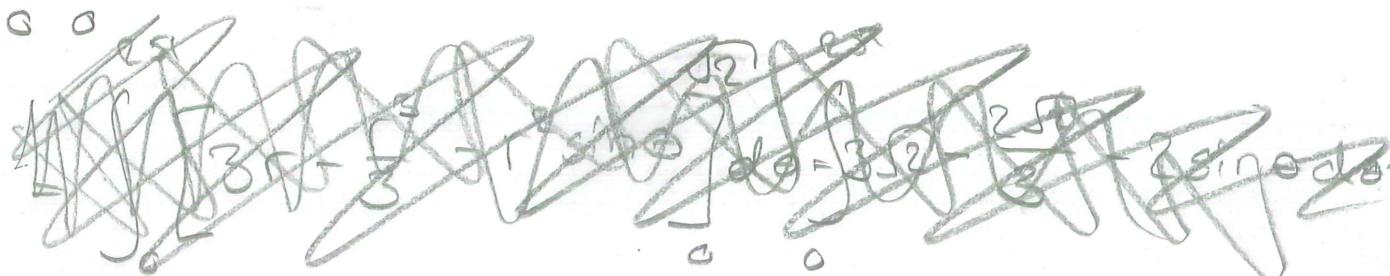


$$\iint_D 1 - 2y - x^2 - y^2 dx dy$$



~~$$\iint_D 1 + r - 2r\sin\theta - r^2 = \begin{cases} x = r\cos\theta \\ y = -1 + r\sin\theta \end{cases}$$~~

$$= \iint_D 3 - r^2 - 2r\sin\theta dr d\theta = \quad dx dy = r dr d\theta$$



3

$$1 - 2y = x^2 + y^2$$

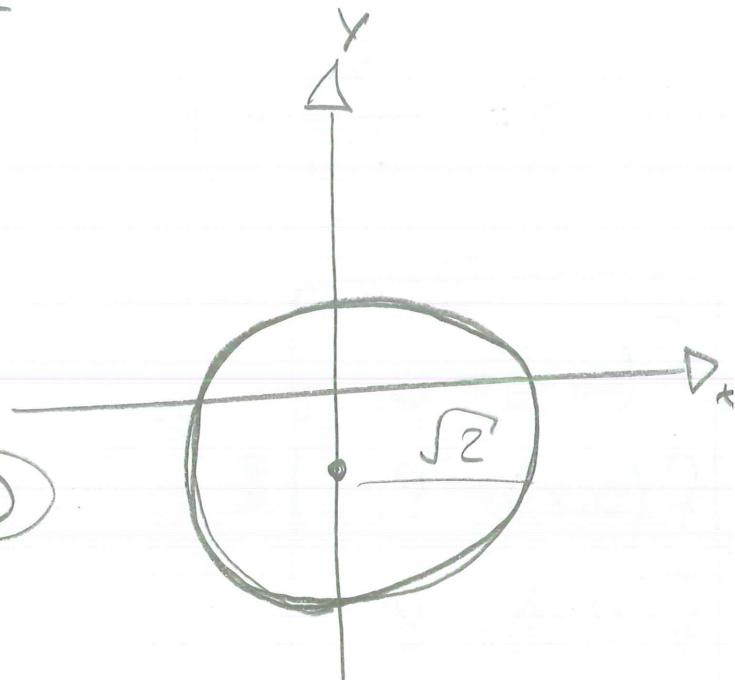
$$\Leftrightarrow x^2 + y^2 + 2y + 1 - 1 = 1$$

$$\Leftrightarrow x^2 + (y+1)^2 - 1 = 1$$

$$\Leftrightarrow x^2 + (y+1)^2 = 2$$

$$\Leftrightarrow \left( \frac{x}{\sqrt{2}} \right)^2 + \left( \frac{y+1}{\sqrt{2}} \right)^2 = 1$$

D



$$\iint_D 1 - 2y - x^2 - y^2 dx dy =$$

$$= \iint_D 1 - 2(-1 + r\sin\theta) - r^2 \cos^2\theta - (r^2 \sin^2\theta) \left\{ \begin{array}{l} x = r\cos\theta \\ y = -1 + r\sin\theta \end{array} \right.$$

$$dx dy = r \cdot dr d\theta$$

$$\iint_D \frac{(3 - 2r\sin\theta - r^2 \cos^2\theta - r^2 \sin^2\theta + 2r\sin\theta)}{r^2} (1) r dr d\theta$$

 $\int_{2\pi}^{2\pi\sqrt{2}}$ 

$$= \int_0^{2\pi} \int_0^{\sqrt{2}} (r - 2r^2 \sin\theta - r^3 + 2r^2 \sin\theta) r dr d\theta$$

$$= 2\pi \int_0^{\sqrt{2}} (2r + r^3) = 2\pi \left[ r^2 + \frac{r^4}{4} \right]_0^{\sqrt{2}} = 2\pi \left( 2 - \frac{4}{4} \right) = \boxed{2\pi}$$

E1

$$f(x, y) = x - xy + y$$

$$y = \frac{9}{x+1} - 1$$

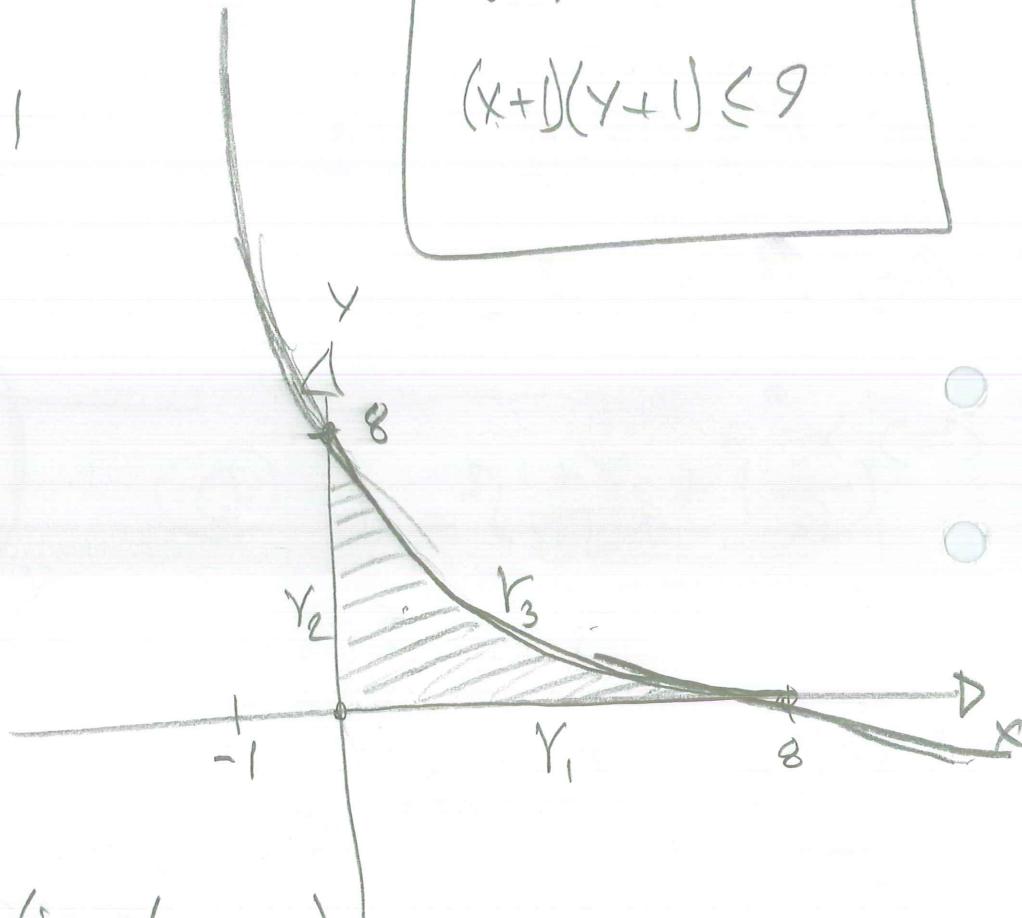
$$f(0, 0) = 0$$

$$f(0, 8) = 8$$

$$f(8, 0) = 8$$

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \end{aligned}$$

$$(x+1)(y+1) \leq 9$$



$$Y_1: f(t, 0) = t \quad (\text{inget max})$$

$$Y_2: f(0, t) = t \quad (\text{inget max})$$

$$Y_3: f\left(t, \frac{9}{t+1} - 1\right) = t - \frac{9t}{t+1} = t + \frac{9}{t+1} - 1$$

$$f'_+ = g \frac{1-t}{1+t} - 1 \Rightarrow f'_+ t = g \frac{-1-t-1+t}{(1+t)^2} = -18 \frac{1}{(1+t)^2} \neq 0$$

När är  $\nabla f = 0$ ?

$$\nabla f = (1-y, 1-x) = (0, 0)$$

$$\Rightarrow \begin{cases} 1-y=0 \\ 1-x=0 \end{cases} \quad (\text{ligger i intervallet})$$

$$f(1,1) = 1-1+1 = \boxed{1}$$

f varierar mellan 0 och 8

$$\begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2$$

$$A = f_{xx}''$$

$$B = f_{yx}''$$

$$C = f_{yy}''$$

$$\Rightarrow \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = \boxed{-1}$$

Indefinit  
sadel

5

$$f''_{xx} + \frac{2}{x} f''_{xy} + \frac{1}{x^2} f''_{yy} = 0, \quad x > 0.$$

$$\begin{cases} c \\ \frac{c}{x} \\ \frac{c}{x^2} \end{cases}$$

$$(f''_{uv} - f''_{vw} \cdot \frac{x}{x^2})$$

$$f'_x = f'_v \cdot \frac{1}{x}$$

$$f'_y = \frac{1}{x} f'_v$$

$$f''_{xx} = f''_{vv} - \left( f''_{uv} - f''_{vw} \cdot \frac{x}{x^2} \right) \frac{x}{x^2} + f'_v \cdot \frac{2}{x^3}$$

$$f''_{xy} = \frac{1}{x} \left( f''_{vu} \cdot 0 + f''_{vw} \cdot \frac{1}{x} \right) = \frac{1}{x^2} f''_{vw}$$

$$f''_{xx} = -\frac{1}{x^2} f'_v + \frac{1}{x} \left( f''_{uv} - f''_{vw} \cdot \frac{x}{x^2} \right)$$

$$f''_{uu} - \frac{2y}{x^2} f''_{uv} + \frac{y^2}{x^4} f''_{ww} + \frac{2y}{x^3} f''_{vv}$$

$$-\frac{2y}{x} \cdot \frac{1}{x^2} f''_{vv} + \frac{2y}{x} \cdot \frac{1}{x} \cdot f''_{vu} - \frac{2y}{x} \cdot \frac{y}{x^3} \cdot f''_{ww}$$

$$+ \frac{y^2}{x^2} \frac{1}{x^2} f''_{ww} = 0$$

NASTAN

(5)

$$f''_{xx} + \frac{2x}{x} f''_{xy} + \frac{x^2}{x^2} f''_{yy} = 0$$

$$\begin{cases} C = x \\ C' = \frac{1}{x} \end{cases}$$

$$f'_x = f'_c \cdot u'_x + f'_v \cdot v'_x = \boxed{f'_c - f'_v \cdot \frac{1}{x^2}}$$

$$f'_y = f'_c \cdot 0 + f'_v \cdot \frac{1}{x} = \boxed{\frac{1}{x} f'_v}$$

$$f''_{xx} = f''_{uu} \cdot u'_x + f''_{uv} \cdot v'_x - f''_{vv} \cdot u'_x \cdot \frac{1}{x^2} + f''_{vv} \cdot v'_x \cdot \frac{1}{x^2} + f'_v \cdot \frac{2x}{x^3} =$$

$$= f''_{uu} - \frac{2x}{x^2} f''_{uv} + \cancel{f''_{uu} \cdot \frac{1}{x^2}} + f'_v \cdot \frac{2x}{x^3}$$

$$f''_{xy} = (f''_{uu} \cdot u'_y + f''_{uv} \cdot v'_y) - (f''_{vu} \cdot u'_x + f''_{vw} \cdot v'_x) \frac{x}{x^2} - f'_v \cdot \frac{1}{x^2}$$

$$\boxed{\frac{1}{x} f''_{uu} - \frac{x}{x^3} f''_{vv} - \frac{1}{x^2} f'_v}$$

$$f''_{xx} = \frac{1}{x} \left( f''_{uu} \circ u_x + f'_{vv} \circ u'_x \right) = \frac{1}{x} \circ \frac{1}{x} f'_{vv} = \boxed{\frac{1}{x^2} f'_{vv}}$$

$$\Rightarrow f_{uu}'' + \cancel{\frac{y^2}{x^4} f_{vv}''} - \frac{2}{x^2} f_{uv}'' + \cancel{\frac{2x}{x^3} f_v'}$$

$$+ \frac{2\gamma}{x^2} f_{uu}'' - \frac{2\gamma^2}{x^4} f_{vv}'' - \frac{2\gamma}{x^3} f_v'$$

$$+ \frac{Y^2}{X^4} f_{vv}^{uu} = f_{uv}^{uu}$$

$$\text{So } f''_{uu} = 0$$

$$\Rightarrow f_c = g(z)$$

$$f(u,v) = u \cdot g(v) + h(v)$$

$$f(x,y) = x g\left(\frac{y}{x}\right) + h\left(\frac{y}{x}\right)$$

b) JA,  $g(t) = e^t$ ,  $h(t) = \cos(t^2)$ .

6

- Gradientens riktning
- a) • Absolutbeloppet av gradienten

Bevis

b)  $f(x,y) = e^{cx}(x - cy^2)$

$$\nabla f = (c e^{cx} + e^{cx}, -e^{cx} \cdot 2cy)$$

$$= (e^{cx}(1+c), -2ce^{cx}y) =$$

$$\nabla f(0,1) = (1+(0-c)c, -2c) = (1-c^2, -2c) = \lambda(4,3)$$

$$\begin{cases} 1-c^2 = 4\lambda \\ -2c = 3\lambda \Rightarrow \lambda = -\frac{2}{3}c \end{cases}$$

$$c^2 - \frac{8}{3}c - 1 = 0$$

$$\begin{cases} c = -\frac{1}{3} \\ c \neq 3 \end{cases}$$

$$\begin{cases} \lambda > 0 \\ c < 0 \end{cases}$$